

Useful logical equivalences

Here is a list of useful logical equivalences to become familiar with. I use \equiv to indicate logical equivalence, as in $A \equiv B$. You shouldn't memorize these, but instead reflect on their meaning (you may use truth tables when possible if it helps you) until you see exactly why they're true and until you remember them naturally. Also, some of them follow easily from others (e.g. the second and third listed under contraposition are just gotten from the first by substituting different things for A and B , and something similar holds for the quantifiers and negation section.).

Double negation

$$\neg\neg A \equiv A$$

$\&$ and \vee (De Morgan's Laws)

$$\neg(A \& B) \equiv \neg A \vee \neg B$$

$$\neg(A \vee B) \equiv \neg A \& \neg B$$

Contraposition

$$A \rightarrow B \equiv \neg B \rightarrow \neg A$$

$$\neg A \rightarrow B \equiv \neg B \rightarrow A$$

$$A \rightarrow \neg B \equiv B \rightarrow \neg A$$

\rightarrow and \vee

$$A \rightarrow B \equiv \neg A \vee B$$

\rightarrow and $\&$

$$A \rightarrow B \equiv \neg(A \& \neg B)$$

Quantifiers and negation (De Morgan's Laws for quantifier logic)

These naturally go in pairs. The first pair is the most basic. The second pair is the effect of a substitution. The third pair then substitutes further.

$$\neg\forall x(\mathcal{A}) \equiv \exists x(\neg\mathcal{A})$$

$$\neg\exists x(\mathcal{A}) \equiv \forall x(\neg\mathcal{A})$$

$$\neg\forall x(\mathcal{A} \rightarrow \mathcal{B}) \equiv \exists x(\mathcal{A} \& \neg\mathcal{B})$$

$$\neg\exists x(\mathcal{A} \& \mathcal{B}) \equiv \forall x(\mathcal{A} \rightarrow \neg\mathcal{B}) \equiv \forall x(\mathcal{B} \rightarrow \neg\mathcal{A})$$

$$\neg\forall x(Ax \rightarrow Bx) \equiv \exists x(Ax \& \neg Bx)$$

$$\neg\exists x(Ax \& Bx) \equiv \forall x(Ax \rightarrow \neg Bx) \equiv \forall x(Bx \rightarrow \neg Ax)$$

Things which are NOT logically equivalent

$$\neg(A \& B) \not\equiv \neg A \& \neg B$$

$$\neg(A \vee B) \not\equiv \neg A \vee \neg B$$

$$A \rightarrow B \not\equiv B \rightarrow A$$

$$A \rightarrow B \not\equiv \neg A \rightarrow \neg B$$

$$\forall x(Ax \rightarrow Bx) \not\equiv \forall x(Ax) \rightarrow \forall x(Bx)$$

$$\exists x(Ax \& Bx) \not\equiv \exists x(Ax) \& \exists x(Bx)$$

etc.