Philosophy 500 — May 19th: Truth Tables

Revised schedule

May 24: Partial truth tables (Ch. 3, sec. 4)
May 26: MIDTERM (no quiz, no homework due)
May 31: MEMORIAL DAY (university holiday)
June 2: Quantified logic: predicates and constants, quantifiers and variables (pp. 48-62)
June 7: Quantified logic: multiple quantifiers (pp. 62-68)
June 9: Quantified logic: syntax and identity (pp. 68-72)
June 14: Practice for final
June 16: FINAL (no quiz, no homework due)

Complete truth tables

Now that we are able to translate some sentences and arguments into a symbolic language, we would like a way to answer logical questions such as "Is this argument valid?", "Is this sentence a tautology?", etc. Using truth-tables is an easy, but sometimes time-consuming, way to do this. We can make a truth table for one or more sentences at a time. But, first, we need to cash out the meaning of the logical operators in terms of truth values (the two possible truth values are true and false).

The truth table for the 'not' operator is very simple. It's based on our definition that $\neg A$ is true if and only if A is false.

A	$\neg A$
T	F
F	T

Likewise, the truth tables for the binary operators follow from the definitions we had for those. Here we're making one truth table for all of them together to save space. Since we now have two sentence letters, and each one can be either true or false, there are four possibilities, so we need four rows.

A	B	$A \mathbin{\&} B$	$A \vee B$	$A \to B$	$A \leftrightarrow B$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Using these, we can build truth tables for more complicated sentences. We proceed from the sentence letters through more and more complicated componenets until we get to the whole sentence. In other words, we're working our way up the tree diagram which represents the sentence. Here's a very explicit table for $((A \to B) \mathbin{\&} B) \vee (B \to \neg A)$

A	B	$A \to B$	$(A \to B) \mathbin{\&} B$	$\neg A$	$B \to \neg A$	$((A \to B) \mathbin{\&} B) \vee (B \to \neg A)$
T	T	T	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T

This table shows that this sentence is a tautology, since it is always true, whatever the truth values of A and of B are.

Writing truth tables this way takes a lot of space and time. So instead we'll use a denser style. We will write the truth value under each letter or operator. To avoid making mistakes when doing this, it's important to be clear about the structure of the sentence (e.g. as illustrated by a tree diagram).

Exercises

1. $A \leftrightarrow [(B \rightarrow A) \,\&\, B]$
2. $[(A \,\&\, B) \,\&\, (B \leftrightarrow C)] \rightarrow (A \,\&\, C)$
3. $[(A \rightarrow B) \rightarrow (B \leftrightarrow A)] \leftrightarrow (B \rightarrow A)$
4. $[(A \vee B) \,\&\, \neg B] \rightarrow (A \leftrightarrow \neg B)$
5. $[(A \,\&\, B) \vee (A \vee \neg B)] \rightarrow [B \,\&\, \neg(A \,\&\, B)]$
6. Is the set $\{A \vee \neg B, A \rightarrow B, \neg(\neg A \vee \neg B)\}$ consistent?
7. Are $\neg[(A \,\&\, B) \vee \neg A]$ and $\neg(A \rightarrow B)$ logically equivalent?
8. Is $A \leftrightarrow [((B \rightarrow A) \& \neg B) \vee \neg(\neg A \vee \neg B)]$ a tautology, a contradiction, or contingent?
9. Is $A \rightarrow \neg B, B \rightarrow A \therefore \neg B \,\&\, \neg A$ valid?
10. Is $A \leftrightarrow \neg(\neg A \,\&\, \neg B)$, $A \vee \neg B \therefore A$ valid?

Relating logical concepts

For each of the follow, state whether it's true or false. If it's true, explain why. If it's false, give an example which shows that.

1. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.

2. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.

3. A and $B \rightarrow A$ aren't logically equivalent, whatever A and B may be.

4. If $A \leftrightarrow B$ is contingent, both A and B are contingent.

5. If $A, B \therefore D$ is valid, so is $A, B \therefore C \vee D$.

Homework #4, Due May 24, 2010

A. Using the key given, translate each of the following sentences. (1 pt. each)

D: Drexel is in Philly.
P: Penn is in Philly.
S: Stan goes to UCLA.
C: Cleo goes to UCLA.

1. Stan goes to UCLA if Cleo does, but Penn and Drexel are in Philly either way.
2. Penn is in Philly only if exactly one out of Stan and Cleo goes to UCLA.
3. Unless Drexel is in Philly, Stan and Cleo don't both go to UCLA.

B. For each of the following, draw a tree diagram if it's a proper sentence. If it's not a proper sentence, write 'NS'. (1 pt. each)

1. $A \,\&\, \neg[(B \vee A) \rightarrow \neg(A \,\&\, \neg\neg C)]$
2. $\neg B \rightarrow [(B \vee A) \,\&\, (A \,\&\, C) \vee B]$
3. $(C \,\&\, \neg D) \leftrightarrow \neg(\neg C \rightarrow \neg B)$

C. Answer each of the following by drawing a **complete** truth table (you should be able to do with one table for each question), and explain briefly how you get the answer from the truth table (e.g. make clear which rows are relevant to your answer and why). Each truth table row is worth half a point, and each answer is worth one point, so the section as a whole is 19 points.

1. Is $(A \vee B) \,\&\, (A \rightarrow \neg B)$ a tautology, a contradiction, or contingent?
2. Is the argument $A \rightarrow B, B \rightarrow C \therefore \neg C \rightarrow \neg A$ valid?
3. Is $A \leftrightarrow \neg B$ logically equivalent to $(A \vee B) \,\&\, (\neg A \vee \neg B)$?

D. For each of the following, state whether it's true or false. If it's true, explain in detail how you can be sure of that. If it's false, give a counterexample (i.e. an example which shows it's false). (2 pts. each)

1. If A is contingent and B is a tautology, then $A \rightarrow B$ is a tautology.
2. If A is contingent and B is a tautology, then $A \leftrightarrow B$ is a contradiction.
3. If $A \leftrightarrow B$ is a contradiction, then A and B are both contradictions.
4. If the set $\{A, \neg B, B \rightarrow A\}$ is consistent, then the argument $A, B \rightarrow A \therefore \neg B$ is valid.
5. Every argument which is sound can be made unsound by adding a premise to it.