

Philosophy 500 — May 10th: Logical concepts

Sentences

For the purposes of this course, we will restrict the word **sentence** to mean **something which can be true or false**. As a result, not everything which we would ordinarily call a sentence will count as a sentence for us. We will refer to true and false as the two possible **truth values**.

Examples of what we will not count as sentences:

Questions: e.g. Is it raining?

Commands: e.g. Don't run with scissors.

Exclamations: e.g. Fire!

Ungrammatical nonsense: e.g. Always Tuesday your Wednesday.

Logical possibility

During this course we will almost always be using the notion of **logical possibility**. We will say that **something is logically possible if it is not contradictory**. This is not the same as being physically possible, biologically possible, likely, etc. **When we speak of something being possible in this course without specifying what kind of possibility we mean, it's logical possibility**. Many things are logically possible which are not true and which we ordinarily wouldn't think of as possible.

Examples. For which of these sentences would it be logically possible to be true?

1. A woman gave birth to an eagle.
2. A woman who was not a woman gave birth to an eagle.
3. The sky is filled with polka dots.
4. Coffee contains caffeine.
5. Gravity pushes everything apart.
6. Bill is neither green nor not green.

Three kinds of sentences

We will divide sentences into three kinds:

Contradictions (or logically false sentences) are ones which could not possibly be true.

Tautologies (or logically true sentences) are ones which could not possibly be false.

Contingent sentences are ones which are neither contradictions nor tautologies. In other words, a sentence is contingent if it's possible for it to be true, and it's possible for it to be false. Or, to put it differently, whether it's true or false is a matter of fact, not a matter of logic.

Examples. For each of the following, say whether it's a tautology, a contradiction, contingent, or not a sentence.

1. Bill Clinton is a chihuahua with wings.
2. Are you satisfied?
3. Either Bill Clinton is a Democrat or he's a Republican.
4. If you cut some corners, some corners will have been cut.
5. Either George Washington lied or he cut down a cherry tree.
6. Either tell me the truth, or don't say anything.
7. There are fifty five states, but there are only ten states.
8. Either the sky is blue or it isn't.

Logical equivalence

Two sentences are logically equivalent to each other if it is impossible for them to have different truth values. In other words, it's not possible for one of them to be true while the other is false.

Examples. Which of the following consist of pairs of logically equivalent sentences?

1. The sky is blue.
Either the sky is blue or it isn't.
2. Bill is a mechanic, and so is Linda.
Bill and Linda are both mechanics.
3. Bill Clinton is both a mechanic and not a mechanic.
Ted Bundy was neither a serial killer nor not a serial killer.
4. Either Alice is tall or she isn't.
Tell me whether Alice is tall or not.
5. Either Alice is tall or she isn't.
Either Bill Clinton is a human or he isn't.

Consistency

A set of sentences is **consistent** if it's possible for all of them to be true at the same time.

Examples. Which of the following sets of sentences are consistent?

1. Ted Bundy is human.
Every human has two dogs.
Ted Bundy doesn't have any dogs.
2. Bill Clinton has no dogs.
Every human has a dog.
3. Bill Clinton is both a dog and not a dog.
Ted Bundy has two dogs.
4. If it's raining, then it's cloudy.
It's raining now.
It's cloudy now.

Arguments

An **argument** is a set of sentences: one or more premises, together with a **conclusion**. The premises are offered as together constituting a reason to believe the conclusion. However, there can be arguments which are so bad that the premises don't actually supply a reason for believing the conclusion. The premises and the conclusion are often identifiable by the use of words such as therefore, so, because, then, thus, etc. The symbol '∴' will be used as shorthand for 'therefore'.

Examples. For each of the following, say whether it contains an argument or not. If it does, identify the premises and the conclusion.

1. Don't lie, because lying is a sin.
2. People shouldn't lie, since lying is a sin.
3. All men are mortal, and Socrates is a man, so Socrates is mortal.
4. It's going to rain tomorrow. When it's raining, it's good to bring an umbrella. Ted has an umbrella.
5. Steve hasn't gone to a football game since 2002. But then he was almost a different person.

Validity and soundness

We will classify arguments according to whether they are valid or not and whether they are sound or not. For this reason, it's very important to not confuse sentences with arguments:

Sentences can be true or false.

Arguments can be valid or invalid, and sound or unsound.

If you say that a sentence is valid, that an argument is true, etc. you are making a mistake, like if you said that a teacup is happy: teacups are not the sort of thing which can be happy or unhappy.

When we say an argument's premises are true, we mean that they're all true. If one or more of them are false, we say its premises are false (even though some of them might be true).

An argument is valid if it's impossible for its premises to be true while its conclusion is false.

An argument is invalid if it's possible for its premises to be true while its conclusion is false.

An argument is sound if it's valid and has true premises.

Whether an argument is valid or not has nothing to do with whether its premises or its conclusion are true or false. It's entirely a question of what's logically possible, not of what's true or false as a matter of fact.

Examples. For each of the following, establish whether it's valid or invalid, and whether it's sound or unsound.

1. Telephones use electricity.

Whatever uses electricity has a battery.

∴ Telephones have batteries.

2. All men are mortal.

Socrates is mortal.

∴ All men are Socrates.

3. Big Ben plays football for the Steelers.

Big Ben doesn't play football.

∴ Big Ben is a clock in England.

4. It doesn't snow in Pittsburgh in July.

If it's snowing then it's below freezing.

∴ The temperature doesn't go below freezing in Pittsburgh in July.

5. Every Democrat voted for Bill Clinton in 1992.

Hilary Clinton was a Democrat in 1992.

Therefore, Hilary Clinton voted for Bill Clinton in 1992.

Working with logical concepts

We have now covered the basic logical concepts and had some practice applying them. But there's more to them than that: you must also be able to successfully **relate them to each other**, and this will be a major part of this course. We will be doing this type of exercise often during the term, and there will be questions like this on homeworks, quizzes, and both exams. So it's very important that you work hard to master this kind of question early.

There is no general method you can mechanically apply to figure these out. Each one requires careful thought and familiarity with the concepts involved. Like almost everything in this course, it's just a case of practice makes perfect.

Examples. For each of the following, establish whether it's true or false. If it's true, explain how you can be sure of that. If it's false, give an example showing that.

1. If an argument is sound, its conclusion must be true.
2. If A and B are both contingent, then they have to be logically equivalent.
3. If $\{A, B\}$ is a consistent set, and C is a tautology, then $\{A, B, C\}$ is also consistent.
4. If an argument is invalid, it must have a false conclusion.
5. If an argument has a conclusion which is a tautology, it must be sound.
6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.

Homework #1, Due May 12, 2010

A. For each of the following, write either 'LT' (logically true), 'LF' (logically false), 'C' (contingent), or 'N' (not a sentence) according to which type it is.

1. Every cat has four legs.
2. Ain't life grand!
3. Whatever you like is liked by you.
4. Obama's dog isn't a dog, but it is Obama's.
5. All trees have giants living inside them.
6. Tim is taller than Jim and Jim is taller than Tim.
7. Every cat is a cat.
8. Every cat dog.

B. For each of the following pairs of sentences, write either 'LE' (logically equivalent) or 'N' (not logically equivalent).

1. Ted Stevens, who is a Democrat, voted for Obama.
Ted Stevens voted for Obama.
2. Mr. Rogers was from Pittsburgh, and Pittsburgh is in Pennsylvania.
Mr. Rogers was from Pennsylvania.
3. Every dog has six legs.
There are no dogs without six legs.
4. Nothing is both a cat and a dog.
Anything which is a cat isn't also a dog.

C. Which of the following sets of sentences are consistent? Write either 'C' for consistent or 'I' for inconsistent.

1. Atlanta is in Houston.
Houston is a state.
Atlanta isn't a city.
2. Bob Dole is a senator with long hair.
Every senator with long hair is a rock star.
Bob Dole isn't a rock star.
3. Bob Dole isn't Bob Dole.
Dole is a fruit company.
4. Either Jack Jones was a singer or he wasn't.
If Jack Jones was a singer, his father was a turtle.

D. For each of the following, state whether it is an argument or not. If it is an argument, list the premises, followed by a ‘ \therefore ’ and the conclusion.

1. This homework is due on Wednesday, so start working on it right now.
2. Because this class goes from 6 to 9:15, it will not be possible to catch every Penguins game in its entirety.
3. Not every dog has its day. Fido is a dog. So Fido will have his day.
4. Unless you watch your step, you might trip and fall. If you do watch your step, you won’t trip and fall. So who wouldn’t want to watch their step?