Philosophy 500 — Practice midterm solutions

A. Relating logical concepts

1. If the set $\{A \vee B, C\}$ is inconsistent, the argument $C \,\&\, A \therefore B$ is valid.

True. If $\{A \vee B, C\}$ is inconsistent, then it's impossible for C to be true while $A \vee B$ is true. But $A \vee B$ is true if and only if one of them true. Therefore, it's impossible for C to be true while either A or B is true. So it's impossible for C and A to both be true. Therefore, it's impossible for $C \,\&\, A$ to be true (since it's true if and only if both C and A are), and so impossible for it to be true while B is false, which means the argument is valid.

2. If A and $B \,\&\, C$ are logically equivalent, then $\{A, B\}$ is consistent.

False. For example, A could be "The sky is both blue and not blue", B could be "The sky is blue", and C could be "The sky isn't blue".

3. If $A \to B$ is a contradiction, then A is a tautology.

True. If $A \to B$ is a contradiction, it means that it's false in every possible world. But $A \to B$ is false if and only if A is true and B is false. Therefore, it would mean that in every possible world A is true and B is false, so A is a tautology, by definition.

4. If $A \vee B$ is contingent, then so is $A \leftrightarrow B$.

False. For example, if A and B are both "The sky is blue", $A \vee B$ is contingent, but $A \leftrightarrow B$ is a tautology.

5. If the argument $A, B \therefore C$ is valid, so is $A \therefore B \to C$.

True. Since the first argument is valid, it's impossible for A and B to be true while C is false. Now, for the second argument to be invalid, it would mean that it's possible for A to be true while $B \to C$ is false. But $B \to C$ is false if and only if B is true and C is false. So what would be required for the second argument to be invalid is that it be possible for A to be true while B is true and C is false. But this is exactly what we know is impossible, since the first argument is valid. So the second argument is also valid.

B. Translations

A: Anna is a pilot.
B: Brock is a pilot.
C: Corrina likes driving.
D: Daisy likes driving.

1. Brock is a pilot only if Anna is, but he's not a pilot unless Daisy likes driving.
   $(B \to A) \,\&\, (\neg B \vee D)$
2. Corrina and Daisy don't both like driving, but one of them does.
   $\neg(C \,\&\, D) \,\&\, (C \vee D)$, or, alternatively, $C \leftrightarrow \neg D$
3. Either Brock isn't a pilot, or it's not the case that both Corrina and Daisy like driving.
   $\neg B \vee \neg(C \,\&\, D)$
4. Corrina likes driving if neither Anna nor Brock is a pilot.
   $(\neg A \,\&\, \neg B) \to C$
5. Unless Corrina likes driving, either Anna is a pilot but Brock isn't, or Daisy doesn't like driving.
   $C \vee (A \,\&\, \neg B) \vee \neg D$

C. Tree diagrams

1. $\neg(A \,\&\, \neg(B \to C))$
   Sentence
2. $B \leftrightarrow \neg(B \vee \neg C)$
   Sentence
3. $A \to (C\neg \,\&\, A)$
   Gibberish
4. $(A \,\&\, B) \,\&\, C \vee B$
   Gibberish (ambiguous)
5. $\neg A \,\&\, (\neg\neg C \to D)$
   Sentence

D. Applying truth tables

1. Is the set $\{A \rightarrow \neg B, B \rightarrow C, A \,\&\, C\}$ consistent?

   It is consistent:

A	B	C	A	$\rightarrow$	$\neg$	B	B	$\rightarrow$	C	A	$\&$	C
T	F	T	T	T	T	F	F	T	T	T	T	T

2. Is the argument $A \rightarrow (A \,\&\, \neg B), \neg B \rightarrow \neg A \therefore \neg(A \vee \neg B)$ valid?

   It isn't valid:

A	B	A	$\rightarrow$	$(A$	$\&$	$\neg$	$B)$	$\neg$	B	$\rightarrow$	$\neg$	A	$\neg$	$(A$	$\vee$	$\neg$	$B)$
F	F	F	T	F	F	T	F	T	F	T	T	F	F	F	T	T	F

3. Are the sentences $A \leftrightarrow (B \rightarrow A)$ and $(B \vee A) \,\&\, \neg(A \,\&\, \neg B)$ logically equivalent?

   They're not logically equivalent:

A	B	A	$\leftrightarrow$	$(B$	$\rightarrow$	$A)$	$(B$	$\vee$	$A)$	$\&$	$\neg$	$(A$	$\&$	$\neg$	$B)$
T	F	T	T	F	T	T	F	T	T	F	F	T	T	T	F

4. Is the sentence $[A \rightarrow (B \vee C)] \leftrightarrow [\neg B \rightarrow (C \,\&\, A)]$ a tautology, a contradiction, or contingent?

   It's contingent:

A	B	C	$[A$	$\rightarrow$	$(B$	$\vee$	$C)]$	$\leftrightarrow$	$[\neg$	B	$\rightarrow$	$(C$	$\&$	$A)]$
T	T	T	T	T	T	T	T	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T	F	T	F	F	T	F	F

5. Is the argument $B \leftrightarrow (A \vee \neg B), A \rightarrow \neg A \therefore A \,\&\, B$ valid?

   It is valid:

A	B	B	$\leftrightarrow$	$(A$	$\vee$	$\neg$	$B)$	A	$\rightarrow$	$\neg$	A	A	$\&$	B
T	T	T	T	T	T	F	T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	F	T	F	F	T	T	F	F
T	T	T	F	F	F	F	T	F	T	T	F	F	F	T
T	T	F	F	F	T	T	F	F	T	T	F	F	F	F

Good luck!