

Philosophy 500 — May 10th: Logical concepts

Logical possibility

Examples. For which of these sentences would it be logically possible to be true? 1, 3, 4, 5

1. A woman gave birth to an eagle.
2. A woman who was not a woman gave birth to an eagle.
3. The sky is filled with polka dots.
4. Coffee contains caffeine.
5. Gravity pushes everything apart.
6. Bill is neither green nor not green.

Three kinds of sentences

Examples. For each of the following, say whether it's a tautology, a contradiction, contingent, or not a sentence.

1. Bill Clinton is a chihuahua with wings.
Contingent.
2. Are you satisfied?
Not a sentence.
3. Either Bill Clinton is a Democrat or he's a Republican.
Contingent.
4. If you cut some corners, some corners will have been cut.
Tautology.
5. Either George Washington lied or he cut down a cherry tree.
Contingent.
6. Either tell me the truth, or don't say anything.
Not a sentence.
7. There are fifty five states, but there are only ten states.
Contradiction.
8. Either the sky is blue or it isn't.
Tautology.

Logical equivalence

Examples. Which of the following consist of pairs of logically equivalent sentences? 2, 3, 5

1. The sky is blue.
Either the sky is blue or it isn't.
2. Bill is a mechanic, and so is Linda.
Bill and Linda are both mechanics.
3. Bill Clinton is both a mechanic and not a mechanic.
Ted Bundy was neither a serial killer nor not a serial killer.
4. Either Alice is tall or she isn't.
Tell me whether Alice is tall or not.
5. Either Alice is tall or she isn't.
Either Bill Clinton is a human or he isn't.

Consistency

Examples. Which of the following sets of sentences are consistent? 2, 4

1. Ted Bundy is human.
Every human has two dogs.
Ted Bundy doesn't have any dogs.
2. Bill Clinton has no dogs.
Every human has a dog.
3. Bill Clinton is both a dog and not a dog.
Ted Bundy has two dogs.
4. If it's raining, then it's cloudy.
It's raining now.
It's cloudy now.

Arguments

Examples. For each of the following, say whether it contains an argument or not. If it does, identify the premises and the conclusion.

1. Don't lie, because lying is a sin.

Not an argument, since "Don't lie" isn't a sentence.

2. People shouldn't lie, since lying is a sin.

Lying is a sin. ∴ People shouldn't lie.

3. All men are mortal, and Socrates is a man, so Socrates is mortal.

All men are mortal. Socrates is a man. ∴ Socrates is mortal.

4. It's going to rain tomorrow. When it's raining, it's good to bring an umbrella. Ted has an umbrella.

No argument (just a series of statements).

5. Steve hasn't gone to a football game since 2002. But then he was almost a different person.

No argument (just a series of statements).

Validity and soundness

Examples. For each of the following, establish whether it's valid or invalid, and whether it's sound or unsound.

1. Telephones use electricity.

Whatever uses electricity has a battery.

∴ Telephones have batteries. *Valid, unsound*

2. All men are mortal.

Socrates is mortal.

∴ All men are Socrates. *Invalid, unsound*

3. Big Ben plays football for the Steelers.

Big Ben doesn't play football.

∴ Big Ben is a clock in England. *Valid, unsound*

4. It doesn't snow in Pittsburgh in July.

If it's snowing then it's below freezing.

∴ The temperature doesn't go below freezing in Pittsburgh in July. *Invalid, unsound*

5. Every Democrat voted for Bill Clinton in 1992.

Hilary Clinton was a Democrat in 1992.

Therefore, Hilary Clinton voted for Bill Clinton in 1992. *Valid, unsound*

Working with logical concepts

Examples. For each of the following, establish whether it's true or false. If it's true, explain how you can be sure of that. If it's false, give an example showing that.

1. If an argument is sound, its conclusion must be true.

True. If an argument is sound, its premises are true (by definition). But if it's sound, it's also valid (by definition), which means that it's impossible for its premises to be true and its conclusion false, and since we had from before that the premises are true, we get that the conclusion is true.

2. If A and B are both contingent, then they have to be logically equivalent.

False. For example, A could be "Bob Dole is a senator" and B could be "Socrates is mortal": they're both contingent, but are not logically equivalent.

3. If $\{A, B\}$ is a consistent set, and C is a tautology, then $\{A, B, C\}$ is also consistent.

True. Because it's a consistent set, that means we can imagine a world in which A and B are both true. But C is a tautology, so that it's true in whatever world we can imagine. Therefore, in that world in which A and B are true, A, B , and C are all true, so $\{A, B, C\}$ is consistent.

4. If an argument is invalid, it must have a false conclusion.

False. For example: "Ghandi was Indian. \therefore Pittsburgh is a city."

5. If an argument has a conclusion which is a tautology, it must be sound.

False. For example: "Ghandi was Russian. \therefore Ghandi was either tall or not tall."

6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.

True. There are 2 possibilities: either A is true, or it's false. If A is true, then since $\{A, B\}$ is inconsistent, we know that B is false. But then this means the argument has a true premise and a false conclusion, which means it's possible for its premise to be true while its conclusion is false, so that it's invalid (by definition) and can't be sound. On the other hand, if A is false, the argument has a false premise, so it's can't be sound either.