Philosophy 500 — May 19th: Solutions to final section

For each of the follow, state whether it's true or false. If it's true, explain why. If it's false, give an example which shows that.

1. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.

First thoughts: *That $A \therefore B$ is valid tells us that if there are any possible worlds in which A is true, B is also true in them. That $\{B, C\}$ is inconsistent tells us that there are no possible worlds in which B and C are both true. Now, the question is whether we can conclude that there's a possible world in which A is true and C false (which would make $A \therefore C$ invalid, or whether it might be the case that there's no such world (i.e. that any possible world in which A is true is one in which C is false). Since we've been prefacing a lot with "if there any possible worlds in which A is true", it's worth considering each case separately. If there is such a world (i.e. if A is not a contradiction), then in that world B would be have to be true. But, then, since $\{B, C\}$ is inconsistent, C would have to be false in that world, which would make $A \therefore C$ invalid. So if A is not a contradiction, we can't have a counterexample. Now we ask about if A is a contradiction.*

Answer: *False. If A and B are both contradictions, for example. For example, we can make A, B, and C all be the same contradictory sentence, e.g. "The sky is both blue and not blue".*

2. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.

First thoughts: *We assume $A \vee B$ is a tautology, which means in any possible world at least one of them is true. Then we ask whether it follows that there's a possible world in which they're both true. One way to go is to look at cases. Obviously, they can't both be contradictions, since then $A \vee B$ couldn't be true. So at least one of them is contingent or a tautology. But if one of them is a tautology, $A \vee B$ is also one, since $A \vee B$ is true whenever either of them. So the question is whether we can then make $\{A, B\}$ inconsistent. If one is a tautology, the only way to do that is to make the other a contradiction.*

Answer: *False. For example: A is "The sky is either blue or not blue" and B is "The sky is both blue and not blue".*

3. A and $B \rightarrow A$ aren't logically equivalent, whatever A and B may be.

First thoughts: *For two sentences to be logically equivalent means that in any possible world in which one of them is true, so is the other, and vice versa, since this excludes there being a world in which one of them is true and the other false but includes both worlds in which both are true and ones in which both are false. Now, in any possible world in which A is true, $B \rightarrow A$ would also be true, by the definition of $\rightarrow$. So A and $B \rightarrow A$ are logically equivalent if and only if in any world in which $B \rightarrow A$ is true, so is A. Again, we can divide into cases. Either B is a tautology, or it's contingent, or it's a contradiction. If it's a tautology, then $B \rightarrow A$ will be true in those worlds (if any) in which A is true, and false in those worlds (if any) in which A is false. So this works.*

Answer: *False. For example, let A be "The sky is blue", and B be "The sky is blue or not blue". Then A and $B \rightarrow A$ are logically equivalent (each is true if and only if the sky is blue).*

4. If $A \leftrightarrow B$ is contingent, both A and B are contingent.

First thoughts: *Suppose $A \leftrightarrow B$ is contingent. That means it's possible for it to be true and possible for it be false. If it's true, one of them is true and the other is false, so we know that this is possible. So they can't both be tautologies, and they can't both contradictions. We want to see whether there is an example in which not both are contingent. So there are two options: neither of them is contingent, or one of them is and the other isn't. If neither of them is contingent, the only case left would be for one of them to be a tautology and the other a contradiction, but then $A \leftrightarrow B$ couldn't be true. So if there is such an example, one has to be contingent and the other not. In fact, any such example works.*

Answer: *False. For example: A can be "The sky is blue" and B can be "The sky is either blue or not blue".*

5. If $A, B \therefore D$ is valid, so is $A, B \therefore C \vee D$.

First thoughts: *This one follows pretty simply from the definitions.*

Answer: *True. That $A, B \therefore D$ is valid means it's impossible for A and B to be true while D is false. For $A, B \therefore C \vee D$ to be valid means that it's impossible for A and B to be true while $C \vee D$ is false. But $C \vee D$ is false if and only if both C and D are false. So for it to be valid means that it's impossible for A and B to be true while C and D are both false. Now, if it were possible for A and B to be true while C and D are both false, it would of course be possible for A and B to be true while D is false, which would contradict the assumption that $A, B \therefore D$ is valid. So it's impossible, and so $A, B \therefore C \vee D$ is valid.* **Note that here I am using proof by contradiction. I showed that if the second argument were invalid, the first one would have to be invalid, which contradicts our assumption. Another way to think about it is the following: We want to show that X logically implies Y, which is to say that it's impossible for X to be true while Y is false. But that's the same as saying that Y being false logically implies X is false. This is completely parallel to the fact that $X \to Y$ is logically equivalent to $\neg Y \to \neg X$. Framing it this way, it is called "proof by contraposition" because $\neg Y \to \neg X$ is the contrapositive of $X \to Y$ (that's what "contrapositive" means).**