Philosophy 500 — May 19th: Partial Truth Tables

In many cases, we don't need to draw a whole truth table in order to obtain our result. For example, if we're trying to find out whether an argument is valid or not, as soon as we get a row in which the premises are all true but the conclusion is false, we know (from the definition of validity) that the argument is invalid. On the other hand, if we keep going row-by-row until we've exhausted all the possibilities without such a row, then we know the argument is valid. But if we were doing an 8-row truth table, and have only done the first 7 rows, it's still possible that the eighth row will turn out to make the premises true and the conclusion false, so we still can't be sure whether the argument is valid or not. Therefore:

One row of a truth table can be enough to show that an argument is invalid, but a complete truth table is required to show that an argument is valid.

Similar considerations apply to other things we want to find out: whether a sentence is a tautology, whether a sentence is a contradiction, whether a sentence is contingent, whether a set of sentences is consistent, whether two (or more) sentences are logically equivalent.

For each of these, figure out (based on the definitions), whether a complete truth table is required, or whether a partial one is enough. If a partial truth table is enough, figure out how many rows it needs to have as a minimum.

To show	We need	To show	We need
Argument valid	complete	Argument invalid	1 row
Sentence a tautology		Sent. not a tautology	
Sentence a contradiction		Sent. not a contradiction	
Sentence contingent		Sent. not contingent	
Sentences consistent		Sent. inconsistent	
Sentences equivalent		Sent. not equivalent	

However, even if we know we only need one row to show that an argument is invalid, we may not know ahead of time whether such a row (i.e. one in which the premises come out true and the conclusion false) exists, or which row it is. One option is to just start doing a complete table, and to stop once we find such a row, since then we've already shown the argument is invalid. But as far as we know, it might be the last row, so we'd end up doing a whole truth table when only one row was necessary. If there are more than 2 variables in our sentences, this could take a long time. So it would be better if there was a way to just get a row that shows the argument is invalid, without having to bother with the others. For this, we'll have to *work backwards, starting from the truth values we're looking for, and then gradually figuring out the truth values we need for the sentence letters.* This is essentially a matter of deduction, combined with dividing into cases, somewhat like playing Sudoku.

Examples

1. Suppose we want to show that the argument: $A \vee B \therefore A \to B$ is invalid. We first draw a truth table, but we don't fill in any of the rows. What we're looking for is a row in which $A \vee B$ is true, and $A \to B$ is false. So we write T under $A \vee B$, and F under $A \to B$. Now we want to fill out the rest, avoiding, as much as we can, making choices. Since we know $A \vee B$ is true, we know that we can't make both A and B false, but we don't know whether we should make A true or B (or both). On the other hand, we also know that $A \to B$ is false, and that can only be the case if A is true and B is false. So we fill in these values, and then finish the table based on that. When we do that, we see that we don't run into any contradictions, so we've successfully found the row.

2. Suppose we want to show that the argument $\neg(A \vee B) \therefore (A \to B) \,\&\, (B \to A)$ is invalid. We start as before, writing T under the $\neg$ in the premise, and F under the & in the conclusion. Now, to make the & false there are 3 possibilities, so we'll avoid that if we can. On the other hand, to make the $\neg$ in the premise true, there's only one: making $A \vee B$ false, so we write F under the $\vee$. And there's only one way $A \vee B$ can be false, which is if A and B are both false, so we fill those in, and we can then finish the table. But when we do that, we see that the conclusion becomes true, contradicting our assumption that it was false. Since we haven't made any choices here, we know that in fact it's impossible to find the row we're looking for: the argument is actually valid. We implicitly have here a proof that it's valid. We can make it explicit by giving the reasoning: Suppose $\neg(A \vee B)$ is true. Then $A \vee B$ must be false. But this means that A and B are both false, in which case $(A \to B) \,\&\, (B \to A)$ is true, as can be shown by a row of its truth table. So the premise logically implies the conclusion.

3. Suppose we want to show that the argument $A \vee B$, $B \vee C \therefore A \leftrightarrow C$ is invalid. Here we see that there's no way to proceed without making a choice at some point. There are three ways for $A \vee B$ to be true, three ways for $B \vee C$ to be true, and two ways for $A \leftrightarrow C$ to be false. So we just have to try one, and if it doesn't work we can then try the other one. So first we can try making A true and C false. Now, since $B \vee C$ had to be true, and C is false, we know we have to make B true. We then finish the row and see that there are no contradictions, so we've shown the argument is invalid.

Exercises

1. Show that the argument "$(A \,\&\, B) \to A \therefore (A \to B) \,\&\, (B \to A)$" is invalid.
2. Show that $(A \to B) \,\&\, (A \vee B)$ is contingent.
3. Show that the argument "$A \to B$, $B \to C$, $C \therefore A$" is invalid.
4. Show that $(B \to A) \vee (A \to B)$ is not logically equivalent to $A \leftrightarrow B$.
5. Show that $(A \vee B) \,\&\, (A \to C)$, $B \to C$, and $\neg C \vee \neg B$ form a consistent set.
6. Show that the argument "$C \to A$, $(A \,\&\, B) \vee (A \leftrightarrow B)$ $\therefore (B \,\&\, C) \,\&\, (A \to B)$" is invalid.
7. Show that the following argument is invalid: "Bob only likes Chinese food if Cindy isn't Chinese. Cindy is Chinese unless Bob likes Chinese food. Therefore, Bob likes Chinese food and Cindy is Chinese".

Relating logical concepts (questions from HW #3 and #4)

For each of the following, state whether it's true or false. If it's true, explain in detail how you can be sure of that. If it's false, give a counterexample (i.e. an example which shows it's false).

1. If $A \rightarrow B$ is true, then $A \therefore B$ is a valid argument.
2. If an argument has a conclusion which is a contradiction, it isn't valid.
3. If A and $\neg B$ are logically equivalent, then $\{A, B\}$ is an inconsistent set.
4. If $\{A, B, C\}$ is a consistent set, then so is $\{A \leftrightarrow B, C\}$
5. If $\neg A \leftrightarrow B$ is contingent, then A and B aren't logically equivalent.
6. If A is contingent and B is a tautology, then $A \rightarrow B$ is a tautology.
7. If A is contingent and B is a tautology, then $A \leftrightarrow B$ is a contradiction.
8. If $A \leftrightarrow B$ is a contradiction, then A and B are both contradictions.
9. If the set $\{A, \neg B, B \rightarrow A\}$ is consistent, then the argument $A, B \rightarrow A \therefore \neg B$ is valid.
10. Every argument which is sound can be made unsound by adding a premise to it.

Homework #5, Due June 2, 2010

A. Answer each of the following by means of a truth table. Try to avoid drawing a complete truth table unless it's necessary. (3 pts. each)

1. Is the sentence $[(A \vee (B \rightarrow C)) \,\&\, (\neg A \rightarrow C)] \rightarrow C$ a tautology, contingent, or a contradiction?

2. Are the sentences $\neg(A \,\&\, B) \vee \neg(B \vee A)$ and $\neg(\neg A \vee \neg B) \rightarrow (\neg A \vee \neg B)$ logically equivalent?

3. Is the argument $A \rightarrow (B \rightarrow C), \neg B \vee \neg C, B \therefore \neg A$ valid?

4. Is the set $\{A \vee \neg B, \neg A \rightarrow D, \neg B \rightarrow \neg A, D \rightarrow (A \,\&\, \neg B)\}$ consistent?

B. For each of the following, state whether it's true or false. If it's true, explain in detail how you can be sure of that. If it's false, give a counterexample (i.e. an example which shows it's false). (2 pts. each)

1. If $A \rightarrow \neg A$ is true, A is a contradiction.

2. If A is a tautology and B is contingent, then $A \leftrightarrow B$ is false.

3. If the set $\{A, B, \neg C\}$ is inconsistent, then $\neg(A \,\&\, B) \vee C$ is true

4. If $A \rightarrow C$ and B are logically equivalent to each other, then the argument $\neg C, B \therefore \neg A$ is valid.