Philosophy 500 — May 17th: More sentential Logic

Gibberish vs proper sentences

Just like in English, the symbols of our formal language can be used to form nonsense expressions. Recall that we have the following symbols which we can use to build sentences: $\rightarrow, \leftrightarrow, \&, \vee, \neg$. We also use parentheses to avoid ambiguity, and capital letters $(A, B, \ldots)$ to stand for English sentences. We need to be able to distinguish between proper sentences and gibberish such as $A((B((\neg \,\&\, C)$.

A proper sentence is basically one that's constructed using the rules of the language, and is completely unambiguous as to its meaning. These rules are:

1. Single letters such as $A, B, C, \ldots$ are sentences.
2. If A is a sentence, so is $\neg A$. ($\neg$ is a unitary operator, meaning it always takes in one sentence).
3. If A and B are sentences, so are $(A \rightarrow B), (A \leftrightarrow B), (A \,\&\, B)$, and $(A \vee B)$. When the meaning is clear, we can leave out the parentheses, but we must remember that this is only for convenience and that they might be necessary later on. Similarly, $A \,\&\, B \,\&\, C$ really is short for $(A \,\&\, B) \,\&\, C$. (These are binary operators, meaning they always take in two sentences).

Example: the expression $A \,\&\, (\neg A \vee B)$ is a proper sentence. To see this, note that $\neg A$ is a proper sentence by rule 2, and then $(\neg A \vee B)$ is a proper sentence by rule 3. And, given that, the whole thing is a proper sentence by rule 3. *What we're doing is looking at how the sentence is made up from smaller parts.*

Example: the expression $A \,\&\, (B \vee \&C)$ is gibberish. There is no rule saying that if B and C are sentences, then $(B \vee \&C)$ is a sentence, or that $\&C$ is a sentence, or that $B\vee$ is a sentence. These symbols just don't go together this way.

Clues that an expression isn't a sentence (these may not be exhaustive):

1. It's ambiguous.
2. It contains two letters next to each other without an operator.
3. It contains two binary operators next to each other without a letter between them.
4. It contains unbalanced parentheses.
5. It contains a $\neg$ that is not followed either by a letter, another $\neg$, or a parenthesis.

Representing sentences as tree diagrams

In order to visualize the structure of a sentence written in the formal language, we can draw a tree diagram corresponding to it. Each of the letters will be represented by a terminal branch (i.e. a leaf). The operators will then be represented by nodes.

Exercises

I. Convert each of the following into symbolic notation, using the given key, and draw a tree diagram to represent it:

A: Art is a grandfather.
B: Blinky is Art's cat.
C: Carol is Art's oldest granddaughter.
D: Disney makes terrible movies.
E: Ernie was on Sesame Street.
F: Floyd is a grandfather.

1. Either Art is a grandfather unless Ernie was on Sesame Street, or Disney makes terrible movies if and only if Blinky is Art's cat.

2. If Disney makes terrible movies, then either Ernie was on Sesame Street or Art is a grandfather.

3. Although Ernie was on Sesame Street, Floyd and Art aren't both grandfathers.

4. If Blinky is Art's cat, then Floyd is a grandfather but Art isn't.

5. Either Floyd isn't a grandfather, or Art isn't, but Disney makes terrible movies either way.

6. It's not the case that Disney doesn't make terrible movies, but it is the case that Disney does make terrible movies and Carol is Art's oldest granddaughter.

7. Disney only makes terrible movies if Blinky is Art's cat, but if Art either is or is not a grandfather, then Disney makes terrible movies.

8. It isn't the case that Floyd is a grandfather or that Ernie was on Sesame Street, but—unless Disney makes terrible movies—Art is a grandfather.

9. Art is a grandfather and either Floyd is one too or Disney makes terrible movies.

II. For each of the following, find out whether it's a proper sentence or gibberish by trying to draw a tree diagram for it.

1. $A \mathbin{\&} (B \rightarrow C) \mathbin{\&} \neg(A \vee B)$
2. $A \rightarrow [A(B \mathbin{\&} C)]$
3. $\neg\neg(A \vee \neg(B \mathbin{\&} \neg D))$
4. $\leftrightarrow (A \mathbin{\&} C) \rightarrow B$
5. $[A \mathbin{\&} (B \vee \neg C)] \rightarrow (A \neg A)$
6. $A \mathbin{\&} B \mathbin{\&} C \vee D$
7. $[(A \rightarrow B) \mathbin{\&} (B \rightarrow C)] \rightarrow (A \rightarrow C)$
8. $A \rightarrow (B \mathbin{\&} (C \neg \mathbin{\&} A))$
9. $(A \mathbin{\&} B) \vee (A \mathbin{\&} \neg B) \rightarrow A$
10. $(C \mathbin{\&} (\neg A \vee BD))$
11. $C \leftrightarrow \neg(\neg A \mathbin{\&} B)$
12. $A \mathbin{\&} \neg(\vee AB) \mathbin{\&} D$
13. $(A \leftrightarrow B) \mathbin{\&} [(\neg A \mathbin{\&} C) \vee \neg D] \mathbin{\&} E$

Relating logical concepts

For each of the follow, state whether it's true or false. If it's true, explain why. If it's false, give an example which shows that.

1. If $A \leftrightarrow B$ is contingent, so is $A \rightarrow B$.

2. If $\{A, B, C\}$ is an inconsistent set, then $\{A, B, \neg C\}$ is a consistent set.

3. If A isn't a proper sentence, neither is $\neg A$.

4. If $A \leftrightarrow B$ is a tautology, so is $\neg A \vee B$.

5. If $A \,\&\, B$ is logically equivalent to $A \vee B$, then A is logically equivalent to B.

6. If $A \vee B$ is contingent, so are A and B.

7. If the argument $A, A \vee B \therefore C$ is sound, so is $A \therefore C$

8. If an argument is valid but not sound, there is a logically possible world in which it would be sound.

Homework #3, Due May 19, 2010

Please answer section A space provided and sections B and C on another piece of paper, and staple them together. Thanks!

A. Using the key given, translate each of the following sentences. (1 pt. each)

G: Gary is afraid of the dark.
S: Sandy is afraid of the dark.
T: Tim is afraid the dark.
N: Nancy knows Jim.
P: Paula knows Jim.

1. If Nancy knows Jim, Sandy and Tim aren't both afraid of the dark.

2. Sandy is afraid of the dark if Tim is, but Gary is either way.

3. Paula only knows Jim if either Gary or Sandy is afraid of the dark.

4. Unless Nancy knows Jim, Paula doesn't.

5. Gary is afraid of the dark if and only if either Nancy or Paula knows Jim.

6. Even though Nancy knows Jim, Paul only knows Jim if Tim is afraid of the dark.

7. Neither Gary nor Tim is afraid of the dark, but if Nancy knows Jim then Tim is.

8. Unless Sandy is afraid of the dark, it isn't the case that Nancy knows Jim or that Greg is afraid of the dark.

9. Either Tim is afraid of the dark, or Gary and Sandy both are.

10. Nancy only knows Jim if Gary is afraid of the dark, but Paula knows Jim regardless.

B. For each of the following, draw a tree diagram if it's a proper sentence. If it's not a proper sentence, write 'NS'.(1 pt. each)

1. $A \to (B \vee C \,\&\, \neg A)$
2. $\neg\neg(A \vee \neg(B \to \neg A))$
3. $\neg(B \to \neg C) \leftrightarrow (B \,\&\, \neg(A \to B))$
4. $(\neg B \,\&\, (C \vee D)) \to C\neg$
5. $\neg B \vee (B \,\&\, \neg C \,\&\, A)$

C. For each of the following, state whether it's true or false. If it's true, explain in detail how you can be sure of that. If it's false, give a counterexample (i.e. an example which shows it's false). (2 pts. each)

1. If $A \to B$ is true, then $A \therefore B$ is a valid argument.
2. If an argument has a conclusion which is a contradiction, it isn't valid.
3. If A and $\neg B$ are logically equivalent, then $\{A, B\}$ is an inconsistent set.
4. If $\{A, B, C\}$ is a consistent set, then so is $\{A \leftrightarrow B, C\}$
5. If $\neg A \leftrightarrow B$ is contingent, then A and B aren't logically equivalent.