Philosophy 500 — June 2: Quantifier logic

Recall from last time:

(1) Some A's aren't B's	(1) $\exists x(Ax \,\&\, \neg Bx)$
(2) Not all A's are B's	(2) $\neg\forall x(Ax \rightarrow Bx)$
(3) All A's are B's	(3) $\forall x(Ax \rightarrow Bx)$
(4) Some A's are B's	(4) $\exists x(Ax \,\&\, Bx)$
(5) No A's are B's	(5) $\neg\exists x(Ax \,\&\, Bx)$
(6) Every A isn't a B	(6) $\forall x(Ax \rightarrow \neg Bx)$

(1) and (2) are logically equivalent to each other, and are the negation of (3).
(5) and (6) are logically equivalent to each other, and are the negation of (4).

More broadly, all of these fall under the two basic patterns of (3) and (4). For example:

"Some A's arent's B's" is just the same as "Some A's are not-B's".

"All A's aren't B's" is just the same as "All A's are not-B's".

Most of the sentences we'll be translating will fall under these two patterns, perhaps with a $\neg$ in front, or can be broken up into pieces which fall under these two patterns. The only complication is that instead of A or B being simple predicates they could be complicated things.

For example:

"Every tall plumber is a turtle who knows Bill" is just:

"All tall plumbers are turtles who know Bill". So we would first write it as:

$\forall x((x \text{ is a tall plumber}) \rightarrow (x \text{ is a turtle who knows Bill}))$

From here it's easy to finish the translation if we have the key.

This is an example of the pattern of (3), in which there is some condition (being an A) and something we affirm of all those things which satisfy the condition (that they are Bs). Sometimes there will be no condition. For example: "Everything is an A", but this is even easier to translate.

The same goes for the pattern of (4): there is a condition (being an A) and something we affirm (being a B), the difference is that here we just affirm this of one of the A's, not all of them.

"Only" and "any"

These are two words to watch out for, in the same way that "only" was one to watch out for in sentential logic. In fact, **"only"** has a similar meaning here.

For example, what does "Only humans can read" mean? The simplest way to rephrase it is: "All readers are humans". We see here a pattern reminiscent of quantifier logic:

$\mathcal{B}$ if $\mathcal{A}$	$\mathcal{A} \rightarrow \mathcal{B}$	All A's are B's	$\forall x(Ax \rightarrow Bx)$
$\mathcal{B}$ only if $\mathcal{A}$	$\mathcal{B} \rightarrow \mathcal{A}$	Only A's are B's	$\forall x(Bx \rightarrow Ax)$

Though it might at first look confusing to pair them this way, it makes sense in that in the first row A is the condition in all four columns. The key is that in both sentential and quantifier logic the "only" flips the order.

What about "Only old dogs like cheese"? This could be interpreted as meaning that "the only things which likes cheese are old dogs", or as "the only dogs which likes cheese are the old ones". But the second interpretation is a matter of context and emphasis: for when the emphasis is placed on "old": "Only *old* dogs like cheese". So we'll interepret it as saying that everything which likes cheese is an old dog.

The word **"any"** is also one to watch out for.
Sometimes "any" indicates a universal quantifier:
Bill is taller than **any** dog
<u>means the same as</u>
Bill is taller than **every** dog.
But sometimes "any" indicates an existential quantifier:
If **anything** is a cat, then Steve is a dog
<u>means the same as</u>
If **something** is a cat, then Steve is a dog.
This is just how this word is used in English, so you just have to pay attention that you capture the meaning of the sentence in your translation.

Meaning and empty predicates

The meaning of sentences such as "All humans are mammals" and so is generally pretty clear. There is one exception, though: when nothing satisfies the condition. For example: "Every five-legged human likes brandy". Should be call this true or false?

In the same way that we said that $\mathcal{A} \rightarrow \mathcal{B}$ is automatically true if $\mathcal{A}$ is false, we will say that "All A's are B's" is automatically true if there are no A's.

In fact, this is already implicit in our saying that (1) and (2) are equivalent:

"Some A's are not-B's" means

"There is something which is A and not-B", so it's false if there are no A's. So if (1) is equivalent to (2), and (2) is the direct negation of (3), then "All A's are B's" must be true if there are no A's, because (1) is false in that case.

Multiple quantifiers

To translate sentences with multiple quantifiers, the best way is to break up the sentence as much as possible, and then proceed in steps.

Examples:

UD: all animals
a: Alice
b: Betty
c: Chris

Ax: x is an ant
Bx: x is a bear
Rx: x is red
Lxy: x is larger than y.
Exy: x could eat y.

1. Every ant is red and every red bear could eat Alice.
   This is just a compound sentence, presenting no special difficulties.
   $\forall x(Ax \to Rx) \,\&\, \forall x((Rx \,\&\, Bx) \to Exa)$
2. Every ant could eat some bear.
   This is not a compound sentence. Here we'll end up with **nested quantifiers**.
   $\forall x(Ax \to (x$ could eat some bear$))$
   Now we just need to translate "x could eat some bear". We need another quantifier, but we need to use a different variable for it:
   $\forall x(Ax \to \exists y(By \,\&\, Exy))$
   The reason we need another variable is because we're within the scope of the first quantifier, so x's meaning is already determined and so we can't apply another x-quantifier.

This is all there is to it, from here on working with multiple quantifiers is just practice. As before, most sentences can be broken up into simpler pieces and where there are quantifiers they usually fit into the two basic patterns (3) and (4).

3. No red ant is larger than every bear which could eat Betty.
   This sounds complicated, but taking it by steps it's not so bad.
   We see it fits pattern (5), which is just the negation of (4).
   $\neg\exists x[(x$ is a red ant$) \,\&\, (x$ is larger than every bear which could eat Betty$)]$
   $\neg\exists x[(Rx \,\&\, Ax) \,\&\, (x$ is larger than every bear which could eat Betty$)]$
   Now we just work on the rest of it. It fits pattern (3).
   $\neg\exists x[(Rx \,\&\, Ax) \,\&\, \forall y((y$ is a bear which could eat Betty$) \to (x$ is larger than $y))]$
   $\neg\exists x[(Rx \,\&\, Ax) \,\&\, \forall y((By \,\&\, Eyb) \to Lxy)]$
4. Some red bear could eat any ant which is larger than some bear.
   The overall sentence fits pattern (4).
   $\exists x((Rx \,\&\, Bx) \,\&\, (x$ could eat any ant which is larger than some bear$))$
   Now this part has the word "any", but we can see it fits pattern (3).
   $\exists x((Rx \,\&\, Bx) \,\&\, \forall y[(y$ is an ant which is larger than some bear$) \to Exy])$
   This part that y is an and that it's larger than some bear.
   $\exists x((Rx \,\&\, Bx) \,\&\, \forall y[(Ay \,\&\, (y$ is larger than some bear$)) \to Exy])$
   The rest fits pattern (4).
   $\exists x((Rx \,\&\, Bx) \,\&\, \forall y[(Ay \,\&\, \exists z(Bz \,\&\, Lyz)) \to Exy])$

Exercises

UD: all people	Bx: x is a bouncer.	Ixyz: x introduced y to z.
a: Alice	Mx: x is a merchant	Kxy: x knows y.
b: Betty	Px: x is a plumber.	Lxy: x likes y.
c: Chris	Tx: x is tall.	Oxy: x is older than y.

1. Anyone who like Alice also likes some bouncer older than Chris.

2. No bouncer whom Betty knows likes anyone whom Alice likes.

3. Only tall plumbers introduced merchants to Betty.

4. Everyone knows someone who knows someone who knows Chris.

5. Some bouncer who's liked by a merchant is older than every plumber.

6. Betty doesn't like any merchants who were introduced to her by a tall plumber.

UD: all people	Bx: x is a bouncer.	Ixyz: x introduced y to z.
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b: Betty	Px: x is a plumber.	Lxy: x likes y.
c: Chris	Tx: x is tall.	Oxy: x is older than y.

7. If any merchant likes every tall bouncer, then Alice knows every plumber.

8. Chris was introduced to everyone he knows.

9. Not everyone whom Alice likes knows someone older than Chris who likes Betty.

10. Some bouncers who where introduced to everyone are older than Betty.

11. No plumber who's tall is older than everyone whom Chris knows.

12. Some merchant likes everyone they know who likes someone older than Alice.

UD: all people	Bx: x is a bouncer.	Ixyz: x introduced y to z.
a: Alice	Mx: x is a merchant	Kxy: x knows y.
b: Betty	Px: x is a plumber.	Lxy: x likes y.
c: Chris	Tx: x is tall.	Oxy: x is older than y.

13. Unless some bouncer knows everyone who likes them, Alice doesn't like any merchants.

14. If any plumber who likes Alice also likes themselves, they are older than Chris.

15. Someone who was introduced to Betty likes every merchant.

16. No tall person likes every plumber, unless that person is older than Alice.

17. Any bouncers who likes everyone they know are older than everyone who likes them.f

18. Alice introduced Betty to all the plumbers she knows.

Homework #7, Due June 9, 2010

A. Translate each of the following sentences in the space provided, using the key given. Most of these are complicated, so I recommend doing the translation in steps on scratch paper first. (1 pt each)

UD: all people	Dx: x is a driver.	Cxyz: x lives closer to y than to z.
k: Karl	Bx: x is a banker.	Axy: x admires y.
j: Jenny	Sx: x is a communist.	Lxy: x has written a letter to y.
d: Debbie	Px: x is pale.	Fxy: x is funnier than y.

1. Karl admires every communist who has written a letter to a banker.

2. No communist who lives closer to Jenny than to Debbie is funnier than every banker.

3. Some driver who is pale admires everyone who's funnier than Jenny.

4. Some bankers are pale, but every communist who's funnier than Karl is pale.

5. Any banker who has written a letter to Debbie is also a communist.

6. Only pale communists admire someone whom Debbie admires.

7. If any communists are pale, Jenny has written a letter to every driver.

8. Karl admires everyone who is funnier than someone who admires every communist.

9. Debbie lives closer to Jenny than any driver who isn't pale does.

UD: all people	Dx: x is a driver.	Cxyz: x lives closer to y than to z.
k: Karl	Bx: x is a banker.	Axy: x admires y.
j: Jenny	Sx: x is a communist.	Lxy: x has written a letter to y.
d: Debbie	Px: x is pale.	Fxy: x is funnier than y.

10. Debbie has written a letter to some communist who doesn't admire any pale bankers.

11. Not every communist is funnier than some driver; but Jenny, who's not a communist, is.

12. Karl and Jenny are both funnier than any communist.

13. There's a pale banker to whom Jenny lives closer than anyone whom Karl admires does.

14. Everyone admires someone, but nobody admires any banker.

15. Anyone who's funnier than some communist is funnier than every banker.

B. For each of the following, state whether it's true or false. If it's true, explain **in paragraph form and in detail** how you can be sure of that. If it's false, give a counterexample (i.e. an example which shows it's false). (2 pts. each)

1. If an argument is valid, its premises can be false while it's conclusion is true.
2. If A and B are logically equivalent, then $\{A, \neg B\}$ is inconsistent.
3. If $\{A, B, C\}$ is inconsistent, then $A \rightarrow \neg(B \vee C)$ is a tautology.
4. If $A \rightarrow B$ is contingent, then $\{A, B\}$ is consistent.
5. If $A \leftrightarrow B$ is contingent, at least one of A or B is contingent.