Philosophy 500 Practice for finding mistakes

What follows is a series of questions about relating logical concepts to each other. After that, each one appears with an answer which contains a mistake. In some cases the first part of the answer (whether the claim is true or false) is right but the example doesn't work or the explanation contains a mistake or fails to show what it's supposed to. In other cases the first part is also wrong. For each of the following, you should identify any mistakes in the proposed answer. After that, I have put the answers: both the answer to what the mistake was, and a correct answer to the question. Note: these are the same questions we've had on the hand-outs, homeworks and the practice and actual midterm, so this can also serve for you as a repository of questions of this type. If you get to the point where you can do all of these easily (it'd be pretty much impossible to memorize them since there are a lot and many of them sound similar to each other but have very different answers) you know you've mastered this type of question.

Examples. For each of the following, establish whether it's true or false. If it's true, explain how you can be sure of that. If it's false, give an example showing that.

1. If an argument is sound, its conclusion must be true.

2. If A and B are both contingent, then they have to be logically equivalent.

3. If $\{A, B\}$ is a consistent set, and C is a tautology, then $\{A, B, C\}$ is also consistent.

4. If an argument is invalid, it must have a false conclusion.

5. If an argument has a conclusion which is a tautology, it must be sound.

6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.

7. If A is a tautology, then $A \,\&\, B$ is a tautology, whatever B is.

8. If $A \therefore B$ is a valid argument, then $A \to B$ is a tautology.

9. If A is a tautology, then the argument $B \therefore A$ is valid, whatever B is.

10. If A is a tautology, and B is contingent, then the argument $A \therefore B$ is invalid.

11. If $A \,\&\, B$ is contingent, then both A and B are contingent.

12. If $A \leftrightarrow B$ is a tautology, then the set $\{A, B\}$ is consistent.

13. If a set of sentences is inconsistent, one of them is a contradiction.

14. If all of an argument's premises are tautologies, but its conclusion isn't, the argument is invalid.

15. If $\{A, B, C\}$ is a consistent set, then the argument $A, B \therefore C$ is valid.

16. If A is a tautology, so is $A \vee B$, whatever B is.

17. If A and B are both contingent, then $A \to B$ is a tautology.

18. If $\neg A$ is contingent, so is A.

19. If an argument has one false premise and one true premise, it's invalid.

20. If an argument is sound and its conclusion is a tautology, its premises are also tautologies.

21. If $A \to B$ is true, then $A \therefore B$ is a valid argument.
22. If an argument has a conclusion which is a contradiction, it isn't valid.
23. If A and $\neg B$ are logically equivalent, then $\{A, B\}$ is an inconsistent set.
24. If $\{A, B, C\}$ is a consistent set, then so is $\{A \leftrightarrow B, C\}$
25. If $\neg A \leftrightarrow B$ is contingent, then A and B aren't logically equivalent.
26. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.
27. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.
28. A and $B \to A$ aren't logically equivalent, whatever A and B may be.
29. If $A \leftrightarrow B$ is contingent, both A and B are contingent.
30. If $A, B \therefore D$ is valid, so is $A, B \therefore C \vee D$.
31. If A is contingent and B is a tautology, then $A \to B$ is a tautology.
32. If A is contingent and B is a tautology, then $A \leftrightarrow B$ is a contradiction.
33. If $A \leftrightarrow B$ is a contradiction, then A and B are both contradictions.
34. If the set $\{A, \neg B, B \to A\}$ is consistent, then the argument $A, B \to A \therefore \neg B$ is valid.
35. Every argument which is sound can be made unsound by adding a premise to it.
36. If $A \to B$ is true, then $A \therefore B$ is a valid argument.
37. If an argument has a conclusion which is a contradiction, it isn't valid.
38. If a valid argument has a true conclusion, it also has true premises.
39. If the argument $A, B \therefore C$ is invalid, $A \to C$ is contingent.
40. If A is logically equivalent to B, $A \to B$ is a tautology.
41. If $\{A, B\}$ is an inconsistent set, $A \to B$ is a contradiction.
42. If $A \vee B$ is a contradiction, A and B are both contradictions too.
43. If the set $\{A \vee B, C\}$ is inconsistent, the argument $C \,\&\, A \therefore B$ is valid.
44. If A and $B \,\&\, C$ are logically equivalent, then $\{A, B\}$ is consistent.
45. If $A \to B$ is a contradiction, then A is a tautology.
46. If $A \vee B$ is contingent, then so is $A \leftrightarrow B$.
47. If the argument $A, B \therefore C$ is valid, so is $A \therefore B \to C$.

Incorrect answers: find and explain everything which is wrong in these answers. In some cases there's more than one error, so, in order to benefit the most from these, don't stop looking just because you've found one. Make sure you're convinced you've found all there is to find.

1. If an argument is sound, its conclusion must be true.
   False. For example: Socrates is a man. Socrates is not a man. Therefore, Socrates is a man.

2. If A and B are both contingent, then they have to be logically equivalent.
   True. If A and B are both contingent, that means they could both be either true or false. Therefore, they have the truth value, and so they're logically equivalent.

3. If $\{A, B\}$ is a consistent set, and C is a tautology, then $\{A, B, C\}$ is also consistent.
   True. If $\{A, B\}$ is consistent, then A and B are both true, and C must be true since it's a tautology. Therefore, $\{A, B, C\}$ is also consistent.

4. If an argument is invalid, it must have a false conclusion.
   False. For example: Bill Clinton is a man. Bill Clinton is Greek. Therefore, Bill Clinton is not a man.

5. If an argument has a conclusion which is a tautology, it must be sound.
   True. Since the conclusion is a tautology, it must always be true. Therefore, there's no way the conclusion could be false, so the argument is sound.

6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.
   False. For example: A is "Bill Clinton is both a man and not a man" and B is "Bill Clinton is a man".

7. If A is a tautology, then $A \,\&\, B$ is a tautology, whatever B is.
   False. For example: A is "Socrates is a man" and B is "Every cat is a cat".

8. If $A \therefore B$ is a valid argument, then $A \rightarrow B$ is a tautology.
   False. For example: A is "It's raining" and B is "It's cloudy".

9. If A is a tautology, then the argument $B \therefore A$ is valid, whatever B is.
   True. Since A is a tautology, it's true. Because A is true, the argument $B \therefore A$ is valid.

10. If A is a tautology, and B is contingent, then the argument $A \therefore B$ is invalid.
    False. For example: A is "Every cat is a cat" and B is "Bill Clinton is human".

11. If $A \,\&\, B$ is contingent, then both A and B are contingent.
    True. Since $A \,\&\, B$ is contingent, that means A and B could be true and could be false. But a sentence being contingent just means that it can be true and it can be false. So A and B are contingent.

12. If $A \leftrightarrow B$ is a tautology, then the set $\{A, B\}$ is consistent.
    True. Since $A \leftrightarrow B$ is a tautology, A and B are both true, which shows they can be true at the same time, so $\{A, B\}$ is consistent.

13. If a set of sentences is inconsistent, one of them is a contradiction.
    False. For example: {Fido is a dog, Fido is a mathematician}.

14. If all of an argument's premises are tautologies, but its conclusion isn't, the argument is invalid.

False. For example: Socrates is either a man or not a man. Therefore, Canada is a country.

15. If $\{A, B, C\}$ is a consistent set, then the argument $A, B \therefore C$ is valid.
    True. Since $\{A, B, C\}$ is consistent, that means it's possible for them all to be true at the same time. But that means it's possible for the argument to have true premises and a true conclusion, so it's valid.

16. If A is a tautology, so is $A \vee B$, whatever B is.
    True. For example: A is "Every dog is a dog" and B is "Some dog is a dog".

17. If A and B are both contingent, then $A \to B$ is a tautology.
    True. If they're contingent, that means they can both be true and they can both be false. But $A \to B$ is false only if A is true and B is false, which isn't possible. So it's a tautology.

18. If $\neg A$ is contingent, so is A.
    False. For example: A is "Every dog is a either a dog or not a dog".

19. If an argument has one false premise and one true premise, it's invalid.
    False. For example: Pittsburgh is a city. Ohio is a city. Therefore, Philadelphia is a city.

20. If an argument is sound and its conclusion is a tautology, its premises are also tautologies.
    True. If an argument is sound, its premises must be true. Therefore, they're tautologies.

21. If $A \to B$ is true, then $A \therefore B$ is a valid argument.
    True. If $A \to B$ is true, that means that A implies B, which means that the argument $A \therefore B$ is valid.

22. If an argument has a conclusion which is a contradiction, it isn't valid.
    False. For example: It's raining. It isn't cloudy. Therefore, it's raining and not raining.

23. If A and $\neg B$ are logically equivalent, then $\{A, B\}$ is an inconsistent set.
    True. Since A and $\neg B$ are logically equivalent, that means A is true and B is false. Therefore $\{A, B\}$ is inconsistent.

24. If $\{A, B, C\}$ is a consistent set, then so is $\{A \leftrightarrow B, C\}$
    False. For example: A is "It's raining", B is "It's warm", C is "Fido is a dog".

25. If $\neg A \leftrightarrow B$ is contingent, then A and B aren't logically equivalent.
    True. If $\neg A \leftrightarrow B$ is contingent, then A and B are contingent, and so they're not logically equivalent.

26. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.
    True. Since $A \therefore B$ is valid, B is true. But since $\{B, C\}$ is inconsistent, B and C can't both be true, so C is false, making $A \therefore C$ invalid.

27. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.
    True. If $\{A, B\}$ was inconsistent, A and B would contradict each other, so $A \vee B$ would be a contradiction.

28. A and $B \to A$ aren't logically equivalent, whatever A and B may be.
    False. For example: A is "It's raining" and B is "It's cloudy".

29. If $A \leftrightarrow B$ is contingent, both A and B are contingent.
True. If $A \leftrightarrow B$ is contingent, it is possible for it to be true. But $A \leftrightarrow B$ is true if A and B are both true or if they're both false. Therefore it's possible for them to both be true and possible for them to both be true, which means that they're both contingent.

30. If $A, B \therefore D$ is valid, so is $A, B \therefore C \vee D$.
False. For example, A is "Fido is a dog", B is "All dogs are mortal", C is "Pittsburgh is a city", and D is "Fido is mortal".

31. If A is contingent and B is a tautology, then $A \rightarrow B$ is a tautology.
False. For example: A is "Al Gore is tall" and B is "Al gore is either tall or not tall".

32. If A is contingent and B is a tautology, then $A \leftrightarrow B$ is a contradiction.
True. Since A is contingent and B is a tautology, they're not logically equivalent, so $A \leftrightarrow B$ has to be false and so it's a contradiction.

33. If $A \leftrightarrow B$ is a contradiction, then A and B are both contradictions.
False. For example: A is "Al Gore is tall" and B is "Al Gore is both tall and not tall".

34. If the set $\{A, \neg B, B \rightarrow A\}$ is consistent, then the argument $A, B \rightarrow A \therefore \neg B$ is valid.
True. Since that set is consistent, it's possible for the premises and the conclusion to all be true at the same time, so the argument is valid.

35. Every argument which is sound can be made unsound by adding a premise to it.
False. For example: "Pittsburgh is a city. Therefore, Pittsburgh is a city" is sound, and nothing you add to it can change that.

36. If $A \rightarrow B$ is true, then $A \therefore B$ is a valid argument.
True. If $A \rightarrow B$ is true, that means A implies B which makes $A \therefore B$ valid.

37. If an argument has a conclusion which is a contradiction, it isn't valid.
False. For example: "Al Gore isn't a man. Therefore, Al Gore is both a man and not a man."

38. If a valid argument has a true conclusion, it also has true premises.
True. By the definition of validity, it's impossible for a valid argument to have a true conclusion and false premises.

39. If the argument $A, B \therefore C$ is invalid, $A \rightarrow C$ is contingent.
True. Since the argument is invalid, it's possible for A to be true while C is false, so $A \rightarrow C$ is contingent.

40. If A is logically equivalent to B, $A \rightarrow B$ is a tautology.
False. For example: A is "Every dog isn't a dog" and B is "No dog is a dog".

41. If $\{A, B\}$ is an inconsistent set, $A \rightarrow B$ is a contradiction.
True. If $\{A, B\}$ is inconsistent, then A and B can't be true at the same time, so if A were true B would be false, which would make $A \rightarrow B$ false. Therefore, $A \rightarrow B$ is a contradiction.

42. If $A \vee B$ is a contradiction, A and B are both contradictions too.
True. Since $A \vee B$ is false if and only if both A and B are both false, its being a contradiction means they're both false. Therefore, they're both contradictions.

43. If the set $\{A \vee B, C\}$ is inconsistent, the argument $C \,\&\, A \therefore B$ is valid.

False. For example: A is "Fido is a dog", B is "Al Gore is a man", and C is "Al Gore is both Fido and not Fido".

44. If A and $B \,\&\, C$ are logically equivalent, then $\{A, B\}$ is consistent.

True. If A and $B \,\&\, C$ are logically equivalent, then A and $B \,\&\, C$ don't contradict each other, so much less do A and B contradict each other. So $\{A, B\}$ is consistent.

45. If $A \to B$ is a contradiction, then A is a tautology.

False. For example: A is "Every dog is a dog" and B is "Some dog is a dog".

46. If $A \vee B$ is contingent, then so is $A \leftrightarrow B$.

False. For example: A is "No dog is a dog" and B is "Pittsburgh is a city".

47. If the argument $A, B \therefore C$ is valid, so is $A \therefore B \to C$.

True. Since the first argument is valid, A, B, and C are true. But then $A \therefore B$ is also true because A and B are true. So the second argument has true premises and a true conclusion, so it's also valid.

Solutions: For each of these questions I first have the incorrect answer from above, then an explanation of what's wrong with it, then a correct answer.

1. If an argument is sound, its conclusion must be true.

   False. For example: Socrates is a man. Socrates is not a man. Therefore, Socrates is a man.

   Errors: That A is true does not mean the argument is valid. An invalid argument can also have a true conclusion.

   Correct answer: True. If an argument is sound, that means it's valid and its premises are true. But being valid means its impossible for its premises to be true while its conclusion is false, which would be the case if the conclusion was false. Therefore, the conclusion is true.

2. If A and B are both contingent, then they have to be logically equivalent.

   True. If A and B are both contingent, that means they could both be either true or false. Therefore, they have the truth value, and so they're logically equivalent.

   Errors: That they both could be either true or false doesn't mean that they have the same truth value. Also, having the same truth value wouldn't mean they're logically equivalent (e.g. "Pittsburgh is a city" and "Ohio is a state" are both true, but aren't logically equivalent).

   Correct answer: False. For example: A is "Pittsburgh is a city" and B is "Ohio is a state".

3. If $\{A, B\}$ is a consistent set, and C is a tautology, then $\{A, B, C\}$ is also consistent.

   True. If $\{A, B\}$ is consistent, then A and B are both true, and C must be true since it's a tautology. Therefore, $\{A, B, C\}$ is also consistent.

   Errors: That the set is consistent doesn't mean A and B are true. Also, even if you were to know they were true, you should give more details than this on how that shows the set is consistent (e.g. appeal to the definition). Generally speaking, the more details the betters.

   Correct answer: True. Since $\{A, B\}$ is consistent, there's a possible world in which A and B are both true. Since C is a tautology, it's true in any possible world, and so it's true in that one. So in that world A, B, and C are all true, which shows it's possible for them to all be true at the same time, so $\{A, B, C\}$ is consistent.

4. If an argument is invalid, it must have a false conclusion.

   False. For example: Bill Clinton is a man. Bill Clinton is Greek. Therefore, Bill Clinton is not a man.

   Errors: This argument does have a false conclusion, so it's not a counterexample (a counterexample would be an argument which is invalid but has a true conclusion).

   Correct answer: False. For example: Bill Clinton is a man. Therefore, Pittsburgh is a city.

5. If an argument has a conclusion which is a tautology, it must be sound.

   True. Since the conclusion is a tautology, it must always be true. Therefore, there's no way the conclusion could be false, so the argument is sound.

   Errors: Having a true conclusion which can't be false doesn't mean the argument is sound. In fact, if the conclusion is a tautology the argument has to be valid, but it could still have a false premise which would mean it's not sound.

   Correct answer: False. For example: "Every cat isn't a cat. Therefore, Every cat is a cat."

6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.

False. For example: A is "Bill Clinton is both a man and not a man" and B is "Bill Clinton is a man".

Errors: This argument isn't sound, so this isn't a counterexample.

Correct answer: True. A sound argument has to have true premises, and since it's valid that means its conclusion is also true (see #1, above). If $\{A, B\}$ is inconsistent, A and B can't both be true. If A is false, the argument can't be sound. On the other hand, if A is true, then B is false, so the argument would be invalid and so not sound either.

7. If A is a tautology, then $A \,\&\, B$ is a tautology, whatever B is.

False. For example: A is "Socrates is a man" and B is "Every cat is a cat".

Errors: A isn't a tautology, so this isn't a counterexample.

Correct answer: False. For example: A is "Socrates is either a man or not a man" and B is "No man is a man".

8. If $A \therefore B$ is a valid argument, then $A \to B$ is a tautology.

False. For example: A is "It's raining" and B is "It's cloudy".

Errors: "It's raining. Therefore it's valid" is not a valid argument. It depends on the fact that it rain only comes from clouds, but this is not a logical fact. So this isn't a counterexample

Correct answer: True. Since $A \therefore B$ is valid, it's impossible for A to be true while B is false. But this is the only way in which $A \to B$ is made false. So it's impossible for $A \to B$ to be false, which means it's a tautology.

9. If A is a tautology, then the argument $B \therefore A$ is valid, whatever B is.

True. Since A is a tautology, it's true. Because A is true, the argument $B \therefore A$ is valid.

Errors: The last sentence is false: that A is true doesn't mean that $B \therefore A$ is valid. For example: "Ohio is a state. Therefore, Pittsburgh is a city" is invalid.

Correct answer: True. Since A is a tautology, it's impossible for A to be false. Therefore, it's impossible for A to be false and B to be true at the same time, which by definition means that $B \therefore A$ is valid.

10. If A is a tautology, and B is contingent, then the argument $A \therefore B$ is invalid.

False. For example: A is "Every cat is a cat" and B is "Bill Clinton is human".

Errors: This argument is invalid, so this isn't a counterexample.

Correct answer: True. Since A is a tautology, it's true in every possible world. On the other hand, B is contingent, so there's a world in which it's false. Therefore, in this world A is true and B is false, so this is possible and the argument is invalid.

11. If $A \,\&\, B$ is contingent, then both A and B are contingent.

True. Since $A \,\&\, B$ is contingent, that means A and B could be true and could be false. But a sentence being contingent just means that it can be true and it can be false. So A and B are contingent.

Errors: That $A \,\&\, B$ is contingent means $A \,\&\, B$ could be true and it could be false. It doesn't mean that A and B could be true and could be false. In fact, it does imply A and B could be true, but this is because if $A \,\&\, B$ is true then A and B are both true. But $A \,\&\, B$ being false wouldn't mean A and B are both false.

Correct answer: False. For example: A is "Every dog is a dog" and B is "Fido is a dog".

12. If $A \leftrightarrow B$ is a tautology, then the set $\{A, B\}$ is consistent.

True. Since $A \leftrightarrow B$ is a tautology, A and B are both true, which shows they can be true at the same time, so $\{A, B\}$ is consistent.

Errors: That $A \leftrightarrow B$ doesn't mean they're both true. In fact, they could be contradictions, or they could be contingent, so long as they're logically equivalent to each other.

Correct answer: False. For example: A is "Every dog isn't a dog" and B is "Every cat isn't a cat".

13. If a set of sentences is inconsistent, one of them is a contradiction.

False. For example: {Fido is a dog, Fido is a mathematician}.

Errors: This set isn't inconsistent (it's not a logical fact that dogs can't be mathematicians), so it's not a counterexample.

Correct answer: False. For example: {Fido is a dog, Fido isn't a dog}.

14. If all of an argument's premises are tautologies, but its conclusion isn't, the argument is invalid.

False. For example: Socrates is either a man or not a man. Therefore, Canada is a country.

Errors: This argument is invalid, so it's not a counterexample.

Correct answer: True. If the conclusion isn't a tautology, it's possible for it to be false. But the premises must always be true. Therefore, it's possible for the conclusion to be true while the conclusion is false, which means the argument is invalid.

15. If $\{A, B, C\}$ is a consistent set, then the argument $A, B \therefore C$ is valid.

True. Since $\{A, B, C\}$ is consistent, that means it's possible for them all to be true at the same time. But that means it's possible for the argument to have true premises and a true conclusion, so it's valid.

Errors: The last sentence is off: validity is about whether or not it's possible to have true premises and a false conclusion, not about whether it's possible to have true premises and a true conclusion. There are invalid arguments with true premises and a true conclusion.

Correct answer: False. For example: "Fido is a dog. Spot is a cat. Therefore, Pittsburgh is a city."

16. If A is a tautology, so is $A \vee B$, whatever B is.

True. For example: A is "Every dog is a dog" and B is "Some dog is a dog".

Errors: $A \vee B$ is a tautology, so this isn't a counterexample.

Correct answer: True. $A \vee B$ is only false if both A and B are false. But A is a tautology, so it can't be false. Therefore, $A \vee B$ can't be false, and it's a tautology.

17. If A and B are both contingent, then $A \rightarrow B$ is a tautology.

True. If they're contingent, that means they can both be true and they can both be false. But $A \rightarrow B$ is false only if A is true and B is false, which isn't possible. So it's a tautology.

Errors: The first sentence is true, but misleading. It's true that they can both be false and they can both be true. But that doesn't mean they can both be true at the same time or both be false at the same time (see the example in the correct answer). Additionally, even if it were the case that they could both be true at the same time and they could both be false at the same time, that wouldn't mean these are the only possibilities and that A can't be true while B is false.

Correct answer: False. For example: A is "Fido is a dog" and B is "Fido isn't a dog".

18. If $\neg A$ is contingent, so is A.

False. For example: A is "Every dog is a either a dog or not a dog".

Errors: A isn't contingent, so this isn't a counterexample.

Correct answer: True. Since $\neg A$ is contingent, it can be true. But $\neg A$ is true only when A is false, so A can be false. Likewise, $\neg A$ can be false, but this is only the case when A is true, so A can be true. Therefore, A is contingent.

19. If an argument has one false premise and one true premise, it's invalid.

False. For example: Pittsburgh is a city. Ohio is a city. Therefore, Philadelphia is a city.

Errors: This argument is invalid, so it's not a counterexample.

Correct answer: False. For example: "Pittsburgh is a city. Ohio is a city. Therefore, Pittsburgh is a city."

20. If an argument is sound and its conclusion is a tautology, its premises are also tautologies.

True. If an argument is sound, its premises must be true. Therefore, they're tautologies.

Errors: This is an example of what we said about not forgetting what a "must" depends on. The first sentence is fine, but the "must" is on the assumption that the argument is sound. In other words, the premises must be true assuming the argument is sound, but that doesn't mean that they must be true as a matter of logic, which is what it means to be a tautology. As an analogy, take the claim "If Barbara is tall, she must not be short". It's true, but it doesn't mean that "Barbara isn't short" is a tautology.

Correct answer: False. For example: "Pittsburgh is a city. Therefore, every city is a city."

21. If $A \rightarrow B$ is true, then $A \therefore B$ is a valid argument.

True. If $A \rightarrow B$ is true, that means that A implies B, which means that the argument $A \therefore B$ is valid.

Errors: This sounds natural, but is a mistake. "A implies B", if not qualified as e.g. "A implies B given the laws of physics", means that A logically implies B. But then this doesn't follow from $A \rightarrow B$ being true, which could happen even if A and B are logically unrelated (see the example in the correct answer).

Correct answer: False. For example: A is "Pittsburgh is a state" and B is "Ohio is a state".

22. If an argument has a conclusion which is a contradiction, it isn't valid.

False. For example: It's raining. It isn't cloudy. Therefore, it's raining and not raining.

Errors: This argument actually isn't valid, so it's not a counterexample.

Correct answer: False. For example: "There is a dog which is not a dog. Therefore, it's raining and not raining."

23. If A and $\neg B$ are logically equivalent, then $\{A, B\}$ is an inconsistent set.

True. Since A and $\neg B$ are logically equivalent, that means A is true and B is false. Therefore $\{A, B\}$ is inconsistent.

Errors: Their being logically equivalent doesn't mean A is true and B is false. It just means that they can't both be true or both be false.

Correct answer: True. Since A and $\neg B$ are logically equivalent, it's impossible for A to be true while $\neg B$ is false. But B is true if and only if $\neg B$ is false. So it's impossible for A and B to both be true, which means the set is inconsistent.

24. If $\{A, B, C\}$ is a consistent set, then so is $\{A \leftrightarrow B, C\}$

False. For example: A is "It's raining", B is "It's warm", C is "Fido is a dog".

Errors: $\{A \leftrightarrow B, C\}$ is consistent, so this is not a counterexample. You might think otherwise because sometimes it's raining without being warm or vice-versa. But this just shows $A \leftrightarrow B$ isn't a tautology, not that it's a contradiction (it's actually contingent).

Correct answer: True. Since $\{A, B, C\}$ is consistent, there's a possible world in which A, B, and C are all true. Now, $A \leftrightarrow B$ is true if A and B are true, so it's true in that possible world. Therefore there's a possible world in which $A \leftrightarrow B$ and C are both true, so the second set is consistent.

25. If $\neg A \leftrightarrow B$ is contingent, then A and B aren't logically equivalent.

True. If $\neg A \leftrightarrow B$ is contingent, then A and B are contingent, and so they're not logically equivalent.

Errors: First of all, that $\neg A \leftrightarrow B$ is contingent doesn't mean A and B are both contingent. Additionally, even if they were both contingent that wouldn't mean they're not logically equivalent.

Correct answer: True, since $\neg A \leftrightarrow B$ is contingent, it's possible for it to be true. But that means it's possible for $\neg A$ and B to have the same truth value. Now, since A always has the opposite truth value to $\neg A$, that means it's possible for A and B to have different truth values, which means they're not logically equivalent.

26. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.

True. Since $A \therefore B$ is valid, B is true. But since $\{B, C\}$ is inconsistent, B and C can't both be true, so C is false, making $A \therefore C$ invalid.

Errors: $A \therefore B$ being valid doesn't mean B is true. Likewise, if C were false, that wouldn't mean $A \therefore C$ is invalid.

Correct answer: False. For example: A is "Some dog isn't a dog", B is "Some cat isn't a cat", and C is "Some man isn't a man".

27. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.

True. If $\{A, B\}$ was inconsistent, A and B would contradict each other, so $A \vee B$ would be a contradiction.

Errors: For two sentences to contradict each other just means they can't both be true at the same time, so the first part is ok. But that doesn't mean $A \vee B$ is a contradiction.

Correct answer: False. For example: A is "Fido is a dog" and B is "Fido isn't a dog".

28. A and $B \rightarrow A$ aren't logically equivalent, whatever A and B may be.

False. For example: A is "It's raining" and B is "It's cloudy".

Errors: A and $B \rightarrow A$ aren't logically equivalent. To see this, notice that it's possible for it to be neither cloudy nor raining. In such a case A would be false but $B \rightarrow A$ would be true.

Correct answer: False. For example: A is "Fido is a dog" and B is "If Fido is a dog then Fido is a dog".

29. If $A \leftrightarrow B$ is contingent, both A and B are contingent.

True. If $A \leftrightarrow B$ is contingent, it is possible for it to be true. But $A \leftrightarrow B$ is true if A and B are both true or if they're both false. Therefore it's possible for them to both be true and possible for them to both be true, which means that they're both contingent.

Errors: All we're entitled to say is that it's possible for both of them to be false or for both of them to be true, not that both these things are possible.

Correct answer: False. For example: A is "All dogs are dogs" and B is "Fido is a dog".

30. If $A, B \therefore D$ is valid, so is $A, B \therefore C \vee D$.

False. For example, A is "Fido is a dog", B is "All dogs are mortal", C is "Pittsburgh is a city", and D is "Fido is mortal".

Errors: The second argument is valid, so this isn't a counterexample.

Correct answer: True. If the first argument is valid, it's impossible for A and B to be true while D is false, so it's also impossible for A and B to be true while C and D are true. But $C \vee D$ is false only if C and D are both false. So it's also impossible for A and B to be true while $C \vee D$ is false, which means the argument is valid.

31. If A is contingent and B is a tautology, then $A \rightarrow B$ is a tautology.

False. For example: A is "Al Gore is tall" and B is "Al gore is either tall or not tall".

Errors: $A \rightarrow B$ is a tautology, so this isn't a counterexample.

Correct answer: True. Since B is a tautology, it's impossible for it to be false. But $A \rightarrow B$ is false if and only if A is true and B is false, so this is also impossible, which means it's a tautology.

32. If A is contingent and B is a tautology, then $A \leftrightarrow B$ is a contradiction.

True. Since A is contingent and B is a tautology, they're not logically equivalent, so $A \leftrightarrow B$ has to be false and so it's a contradiction.

Errors: That they're not logically equivalent doesn't mean $A \leftrightarrow B$ is false, much less than it's a contradiction.

Correct answer: False. For example: A is "Fido is a dog" and B is "Every dog is a dog".

33. If $A \leftrightarrow B$ is a contradiction, then A and B are both contradictions.

False. For example: A is "Al Gore is tall" and B is "Al Gore is both tall and not tall".

Errors: $A \leftrightarrow B$ isn't a contradiction, so this isn't a counterexample.

Correct answer: False. For example: A is "Fido is a dog" and B is "Fido isn't a dog".

34. If the set $\{A, \neg B, B \rightarrow A\}$ is consistent, then the argument $A, B \rightarrow A \therefore \neg B$ is valid.

True. Since that set is consistent, it's possible for the premises and the conclusion to all be true at the same time, so the argument is valid.

Errors: Validity is not about whether it's possible for the premises and the conclusion to be true at the same time.

Correct answer: False. For example: A is "Every dog is a dog" and B is "Fido is a dog".

35. Every argument which is sound can be made unsound by adding a premise to it.

False. For example: "Pittsburgh is a city. Therefore, Pittsburgh is a city" is sound, and nothing you add to it can change that.

Errors: Adding "Pittsburgh isn't a city" would make it sound.

Correct answer: True. Adding "Some dog isn't a dog" to any sound argument would make it unsound.

36. If $A \to B$ is true, then $A \therefore B$ is a valid argument.

True. If $A \to B$ is true, that means A implies B which makes $A \therefore B$ valid.

Errors: That $A \to B$ is true doesn't mean A implies B. For that you'd need $A \to B$ to be true as a matter of logic (i.e. a tautology).

Correct answer: False. For example: A is "Pittsburgh is a city" and B is "Ohio is a state".

37. If an argument has a conclusion which is a contradiction, it isn't valid.

False. For example: "Al Gore isn't a man. Therefore, Al Gore is both a man and not a man."

Errors: This argument isn't valid, so it's not a counterexample.

Correct answer: False. For example: "No man is a man. Therefore, Al Gore is both a man and not a man".

38. If a valid argument has a true conclusion, it also has true premises.

True. By the definition of validity, it's impossible for a valid argument to have a true conclusion and false premises.

Errors: This gets the definition of validity backwards.

Correct answer: False. For example: "All humans are men. Al Gore is a human. Therefore, Al Gore is a man."

39. If the argument $A, B \therefore C$ is invalid, $A \to C$ is contingent.

True. Since the argument is invalid, it's possible for A to be true while C is false, so $A \to C$ is contingent.

Errors: This just shows it's possible for $A \to C$ to be false. But for it to be contingent, it also has to be possible for it to be true.

Correct answer: False. For example: A is "Every dog is a dog", B is "Pittsburgh is a city", C is "Some dog isn't a dog".

40. If A is logically equivalent to B, $A \to B$ is a tautology.

False. For example: A is "Every dog isn't a dog" and B is "No dog is a dog".

Errors: $A \to B$ is a tautology, so this isn't a counterexample.

Correct answer: True. Since A is logically equivalent to B, it's impossible for them to have different truth values. So it's impossible for A to be true and B false, which is what it would take to make $A \to B$ false. So $A \to B$ can't be false, which means it's a tautology.

41. If $\{A, B\}$ is an inconsistent set, $A \to B$ is a contradiction.

True. If $\{A, B\}$ is inconsistent, then A and B can't be true at the same time, so if A were true B would be false, which would make $A \to B$ false. Therefore, $A \to B$ is a contradiction.

Errors: It's true that if A were true then $A \to B$ would be false. But that doesn't mean $A \to B$ is a contradiction, since it leaves open the possibility that A might be false, in which case $A \to B$ would be true.

Correct answer: False. For example: A is "Some dog isn't a dog" and B is "Some cat isn't a cat".

42. If $A \vee B$ is a contradiction, A and B are both contradictions too.

True. Since $A \vee B$ is false if and only if both A and B are both false, its being a contradiction means they're both false. Therefore, they're both contradictions.

Errors: Just because A and B are false, doesn't mean they're contradictions.

Correct answer: True. If $A \vee B$ is a contradiction, it's impossible for it to be true. But it's true unless A and B are both false. So it's impossible for either of them to be true, which means they're both contradictions.

43. If the set $\{A \vee B, C\}$ is inconsistent, the argument $C \,\&\, A \therefore B$ is valid.

False. For example: A is "Fido is a dog", B is "Al Gore is a man", and C is "Al Gore is both Fido and not Fido".

Errors: The argument is valid, so this isn't a counterexample.

Correct answer: True. Since that set is inconsistent, it's impossible for C and $A \vee B$ to both be true, which is to say, for C to be true while either A or B is true. In particular, it's impossible for C to be true while A is true. Therefore, it's impossible for $C \,\&\, A$ to be true, so it's impossible for $C \,\&\, A$ to be true while B is false, which means the argument is valid.

44. If A and $B \,\&\, C$ are logically equivalent, then $\{A, B\}$ is consistent.

True. If A and $B \& C$ are logically equivalent, then A and $B \& C$ don't contradict each other, so much less do A and B contradict each other. So $\{A, B\}$ is consistent.

Errors: Two contradictions are logically equivalent even though they contradict each other (a contradiction contradicts itself as well as everything else).

Correct answer: False. For example: A is "Fido is both a dog and not a dog", B is "Fido is a dog" and C is "Fido is not a dog".

45. If $A \rightarrow B$ is a contradiction, then A is a tautology.

False. For example: A is "Every dog is a dog" and B is "Some dog is a dog".

Errors: A is a tautology, so this isn't a counterexample.

Correct answer: True. If $A \rightarrow B$ is a contradiction, then it must be false in any possible world. But it's false if and only if A is true and B is false. So A is true in any possible world, which means it's a tautology.

46. If $A \vee B$ is contingent, then so is $A \leftrightarrow B$.

False. For example: A is "No dog is a dog" and B is "Pittsburgh is a city".

Errors: $A \leftrightarrow B$ is contingent, so this isn't a counterexample.

Correct answer: False. For example: A is "Fido is a dog" and B is "Fido is a dog and the sky is either blue or not blue".

47. If the argument $A, B \therefore C$ is valid, so is $A \therefore B \rightarrow C$.

True. Since the first argument is valid, A, B, and C are true. But then $A \therefore B$ is also true because A and B are true. So the second argument has true premises and a true conclusion, so it's also valid.

Errors: The argument being valid doesn't mean its premises or its conclusion are true. Likewise, having true premises and a true conclusion doesn't mean an argument is valid.

Correct answer: True. If the first argument is valid, it's impossible for A and B to be true while C is false. But this is the same as saying it's impossible for A to be true while B is true and C is false. And $B \rightarrow C$ is false if and only if B is true and C is false. So it's impossible for A to be true while $B \rightarrow C$ is false, which means the second argument is valid.