Philosophy 500 — Midterm Solutions

A. Relating logical concepts: For each of the following, state whether it's true or false, and either explain **in full detail and in paragraph form** why it's true or give an example to show that it's false. **Note**: If you give an example which works, you will get full credit. On the other hand, if you give an example that doesn't work, it might be good to have something written about it to help me know what had in mind when I'm assigning any partial credit (also: *trying to explain why it works might help you realize it doesn't work*). (40 points).

1. If a valid argument has a true conclusion, it also has true premises.

   False. For example: Bill Clinton is both human and not human. Therefore, Bill Clinton is human.

2. If the argument $A, B \therefore C$ is invalid, $A \rightarrow C$ is contingent.

   False. We know it's possible for A and B to be true while C is false. This would mean $A \rightarrow C$ can be false. So if it's going to not be contingent, it has to be a contradiction, which means that it's impossible for it to be true. It'd be true if A were false or if C were true, so A needs to be a tautology and C a contradiction for our counterexample. The only other restraint is that the argument needs to be invalid, so we need to make sure that B can be true. For example: A is "Socrates is either a man or not a man", B is "Socrates is a man", and C is "Socrates is both a man and not a man".

3. If A is logically equivalent to B, $A \rightarrow B$ is a tautology.

   True. If A and B are logically equivalent, it's impossible for them to have different truth values. But $A \rightarrow B$ is, by definition, false if and only if A is true and B false, which is a case of their having different truth values. So it's impossible for $A \rightarrow B$ to be false, which means it's a tautology.

4. If $\{A, B\}$ is an inconsistent set, $A \rightarrow B$ is a contradiction.

   False. We want it so that $A \rightarrow B$ can be true, in order for it not to be a contradiction. Now, it is true if A is false or if B is true. So there are several ways we could go. One is to make A a contradiction, which will both make the set inconsistent and make $A \rightarrow B$ a tautology. For example, we can make A be "Socrates is both a man and not a man" and B be "Socrates is a man".

5. If $A \vee B$ is a contradiction, A and B are both contradictions too.

   True. Since $A \vee B$ is a contradiction, it's false in every possible world. But $A \vee B$ is false if and only if A and B are false. So that means that in every possible world both A and B are false. So in every possible world A is false, which means it's a contradiction, and in every possible world B is false, which means it's a contradiction too.

B. Translations: Convert each of the following sentences into symbolic form, preserving its logical structure, using the key given below (15 points).

A: Andy likes gorgonzola.
B: Betty likes gorgonzola.
C: Cathy is a chef.
D: Dirk is a chef.

1. Either Andy or Betty likes gorgonzola, but they don't both like it.
   $(A \vee B) \,\&\, \neg(A \,\&\, B)$
2. Cathy is a chef if Dirk is a chef, but Dirk is a chef only if Andy likes gorgonzola.
   $(D \rightarrow C) \,\&\, (D \rightarrow A)$
3. Unless Andy likes gorgonzola, either Cathy or Dirk is a chef.
   $A \vee C \vee D$
4. If Andy likes gorgonzola, it isn't the case that Cathy and Dirk are both chefs.
   $A \rightarrow \neg(C \,\&\, D)$
5. Cathy is a chef, but Dirk is only a chef if Betty and Andy both like gorgonzola.
   $C \,\&\, (D \rightarrow (B \,\&\, A))$

C. Tree diagrams: Draw a tree diagram for each of the following which are proper sentences. For those which aren't proper sentences, write 'Gibberish' instead. (15 points).

1. $\neg[\neg(A \,\&\, \neg B) \rightarrow \neg C]$
2. $(A \,\&\, \neg B) \,\&\, \neg C \rightarrow (A \,\&\, B)$
   Gibberish
3. $\neg B \vee \neg[(A \,\&\, \neg C) \leftrightarrow \neg B]$
4. $(A \,\&\, \neg B) \rightarrow [A \,\&\, (B \neg C)]$
   Gibberish
5. $A \rightarrow \neg[(A \,\&\, \neg\neg C) \leftrightarrow \neg B]$

D. Applying truth tables: Answer each of the following, showing your answer is right by means of a truth table (partial or complete, but I recommend avoiding doing a complete table unless it's necessary). Make explicit which rows (or all of them if that's the case) yielded your answer. (30 points).

1. Is the set $\{\neg A \rightarrow \neg B, \neg(A \,\&\, B), \neg B \rightarrow (A \,\&\, B)\}$ consistent?
   No, it's inconsistent (requires a full table to show).

A	B	$\neg$	A	$\rightarrow$	$\neg$	B	$\neg$	$(A$	$\&$	$B)$	$\neg$	B	$\rightarrow$	$(A$	$\&$	$B)$
T	T	F	T	T	F	T	F	T	T	T	F	T	T	T	T	T
T	F	F	T	T	T	F	T	T	F	F	T	F	F	T	F	F
F	T	T	F	F	F	T	T	F	F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T	F	F	F	T	F	F	F	F	F

2. Is the argument $A \leftrightarrow B, C \rightarrow \neg B \therefore \neg C \vee B$ valid?

No, it's invalid. There is only one row which works to show this:

A	B	C	A	$\&$	B	C	$\rightarrow$	$\neg$	B	$\neg$	C	$\vee$	B
F	F	T	F	T	F	T	T	T	F	F	T	F	F

3. Are the sentences $(A \vee \neg B) \,\&\, (C \vee A)$ and $(B \leftrightarrow A) \,\&\, (\neg A \rightarrow C)$ logically equivalent?

No, they're not. There are two rows either of which works to show this:

A	B	C	$(A$	$\vee$	$\neg$	$B)$	$\&$	$(C$	$\vee$	$A)$	$(B$	$\leftrightarrow$	$A)$	$\&$	$(\neg$	A	$\rightarrow$	$C)$
T	F	T	T	T	T	F	T	T	T	T	F	F	T	F	F	T	T	T
T	F	F	T	T	T	F	T	F	T	T	F	F	T	F	F	T	T	F

4. Is the sentence $[A \leftrightarrow (A \& \neg B)] \rightarrow \neg(A \& B)$ a tautology, a contradiction, or contingent?

It's a tautology. Showing this requires a complete table:

A	B	$[A$	$\leftrightarrow$	$(A$	$\&$	$\neg$	$B)]$	$\rightarrow$	$\neg$	$(A$	$\&$	$B)$
T	T	T	F	T	F	F	T	T	F	T	T	T
T	F	T	T	T	T	T	F	T	T	T	F	F
F	T	F	T	F	F	F	T	T	T	F	F	T
F	F	F	T	F	F	T	F	T	T	F	F	F

5. Is the argument $A \rightarrow \neg B, \neg(B \,\&\, \neg A) \therefore A \,\&\, \neg B$ valid?

No, it's not valid. There's only one row that works to show this:

A	B	A	$\rightarrow$	$\neg$	B	$\neg$	$(B$	$\&$	$\neg$	$A)$	A	$\&$	$\neg$	B
F	F	F	T	T	F	T	F	F	T	F	F	F	T	F