Philosophy 500 — May 12th: Sentential Logic

E.g. #1. Using the key given, translate each of the following sentences.

A: Alice voted for Lincoln.
B: Bob voted for Lincoln.
E: Lincoln got elected.
P: Lincoln became president.

(a) Lincoln got elected even though neither Alice nor Bob voted for him.
$E \,\&\, \neg A \,\&\, \neg B$
(b) Lincoln only became president if Alice voted for him.
$P \rightarrow A$
(c) Unless Lincoln got elected or Alice voted for him, he didn't become president.
$E \vee A \vee \neg P$
(d) Alice voted for Lincoln only if he got elected and became president.
$A \rightarrow (E \,\&\, P)$
(e) Alice and Bob didn't both vote for Lincoln.
$\neg(A \,\&\, B)$

E.g. #2. Using the key given, translate each of the following sentence.

A: Alex is an astronaut.
B: Betty is an astronaut.
C: Cindy is an astronaut.
P: Pittsburgh is in Russia.
T: Toronto is in Russia.

(a) Toronto and Pittsburgh aren't both in Russia.
$\neg(T \,\&\, P)$
(b) If Alex and Betty are both astronauts, so is Cindy.
$(A \,\&\, B) \rightarrow C$
(c) Toronto is in Russia unless neither Cindy nor Alex is an astronaut.
$T \vee (\neg C \,\&\, \neg A)$
(d) Toronto is in Russia if and only if it isn't the case that both Betty and Cindy are astronauts.
$T \leftrightarrow \neg(B \,\&\, C)$
(e) Alex is an astronaut only if Pittsburgh and Toronto are both in Russia.
$A \rightarrow (P \,\&\, T)$
(f) If Alex isn't an astronaut, neither is Betty, but Cindy is an astronaut either way.
$(\neg A \rightarrow \neg B) \,\&\, C$
(g) Unless Toronto is in Russia, Cindy is an astronaut but Alex isn't.
$T \vee (C \,\&\, \neg A)$
(h) If any out of Alex, Betty, and Cindy is an astronaut, Pittsburgh is in Russia.
$(A \vee B \vee C) \rightarrow P$
(i) If Alex is an astronaut and so is Betty, Toronto and Pittsburgh aren't both in Russia.
$(A \,\&\, B) \rightarrow \neg(T \,\&\, P)$
(j) Alex and Betty are both astronauts if and only if neither of them is.
$(A \,\&\, B) \leftrightarrow (\neg A \,\&\, \neg B)$
(k) If Betty is an astronaut, then Toronto isn't in Russia unless Pittsburgh is.
$B \rightarrow (\neg T \vee P)$
(l) Either Toronto is in Russia or Pittsburgh isn't.
$T \vee \neg P$
(m) Unless Alex is an astronaut, neither Betty nor Cindy is.
$A \vee \neg(B \vee C)$

Relating logical concepts

For each of the follow, state whether it's true or false. If it's true, explain why. If it's false, give an example which shows that.

1. If A is a tautology, then $A \,\&\, B$ is a tautology, whatever B is.

False. For example if A is "Pittsburgh is a city or isn't a city" and B is "Pittsburgh is a state", $A \,\&\, B$ is contingent.

2. If $A \therefore B$ is a valid argument, then $A \rightarrow B$ is a tautology.

True. Since it's a valid argument, it's impossible for A to be true while B is false. But $A \rightarrow B$ is only false if A is true and B is false. Therefore, it's not possible for $A \rightarrow B$ to be false, which means it's a tautology.

3. If A is a tautology, then the argument $B \therefore A$ is valid, whatever B is.

True. Since A is a tautology, it's impossible for it to be false. Therefore, whatever B is, it's impossible for B to bet true and A false.

4. If A is a tautology, and B is contingent, then the argument $A \therefore B$ is invalid.

True. Since B is contingent, there is some possible world in which is false. And since A is a tautology, it's true in any possible world, so in particular it's true in that world in which B is false. Therefore, in that world B is false and A is true, which shows that it's possible for the argument's premise to be true while its conclusion is false, meaning it's invalid.

5. If $A \,\&\, B$ is contingent, then both A and B are contingent.

False. One of them could be a tautology. For example, if A and B are as in #1 above, then $A \,\&\, B$ is contingent even though A is a tautology.

6. If $A \leftrightarrow B$ is a tautology, then A and B are logically equivalent.

True. If it's a tautology, that means that in any possible world either both A and B are true, or both A and B are false, since these are the cases in which $A \leftrightarrow B$ is true. Therefore there's no possible world in which one of them is true and the other false, so (by definition) they're logically equivalent.

<u>Answers to exercise D from Ch. 1 of the book.</u> Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument that has one false premise and one true premise.

Possible. For example: Every Democrat is a Republican. Bill Clinton is a Democrat. Therefore, Bill Clinton is a Republican.

2. A valid argument that has a false conclusion.

Possible. The same example as in #1 works here.

3. A valid argument the conclusion of which is a contradiction.

Possible. This is only the case if the premises are contradictory. For example: Fido is a dog. Fido isn't a dog. Therefore, Fido both is and isn't a dog.

4. An invalid argument, the conclusion of which is a tautology.

Not possible. If the conclusion is a tautology, it's impossible for the conclusion to be false. But then it's impossible for the conclusion to be false and the premises true, so by definition the argument is valid.

5. A tautology that is contingent.

Not possible. A contingent sentence is one that is neither a tautology or a contradiction, so it's clear. Alternatively, a contingent sentence is one which could be true and could be false. Since it could be false, it's (by definition) not a tautology.

6. Two logically equivalent sentences, both of which are tautologies.

Possible—in fact, all tautologies are logically equivalent to each other since it's impossible for any of them to ever be false. E.g. Fido either is or isn't a dog. Bingo either is or isn't a dog.

7. Two logically equivalent sentences, one of which is a tautology and one of which is contingent.

Not possible. Since one of them is contingent, there's a possible world in which it's false. But the other one is a tautology, so it's true in any possible world, including that one. So in that world the tautology is true and the contigent sentence is false, which shows they're not logically equivalent.

8. Two logically equivalent sentences that together are an inconsistent set.

Possible—any two contradictions. For example: Fido is both a dog and not a dog. Bingo is both a dog and not a dog.

9. A consistent set of sentences that contains a contradiction.

Not possible. A contradiction can't be true, so any set containing it will always have at least one false sentence (that contradiction) and can't be consistent.

10. An inconsistent set of sentences that contains a tautology.

Possible. For example {*Fido is either a dog or not a dog, Fido is both a dog and not a dog*}.