Philosophy 500 — Practice final solutions, June 10

Note: For sections A and B I give more than is needed to answer the question (in A I explain why and in B I give intermediate steps). This is just for your benefit in comparing the solution to what you've done, you don't need to do that on the final.

UD: all animals	Tx: x is a tiger	Lxyz: x likes y better than z
a: Alyssa	Bx: x is a bird.	Axy: x is afraid of y.
b: Brad	Fx: x has feathers.	Cxy: x is cuter than y.
e: Ella	Hx: x has hair.	Sxy: x has sniffed y.

A. Quantifier logic syntax: For each of the following, write 'S' if it's a both a proper sentence and a proper formula, 'F' if it's a proper formula but not a proper sentence, and 'N' if it's neither a proper formula or a proper sentence. (10 points).

1. $\forall x(Cxa \rightarrow \neg Lyex)$
   F (y appears free)
2. $\neg\exists b(Tb \,\&\, \forall y(Layb))$
   N (a quantifier has to be paired with a variable, not a constant)
3. $\forall y(Ty \,\&\, Ayx \rightarrow Say)$
   N (ambiguous)
4. $\neg\exists x[(Abx \vee Hx) \,\&\, \forall z(Lzbx)]$
   S
5. $Bx \,\&\, \forall x(Ax \rightarrow Fx)$
   N (A needs two inputs)
6. $\forall y(Ayy \rightarrow \neg\exists x(Lxye \vee Sbx)) \vee Aae$
   S
7. $Fx \rightarrow Te$
   F (x appears free)
8. $\forall y[(Cay \vee y) \rightarrow Hx]$
   N (only proper formulas can be joined together with $\vee$, and y isn't a proper formula)
9. $\exists x(\neg x = a \,\&\, (Fx \vee Aex))$
   S
10. $\forall y((Hy \,\&\, Ty) \rightarrow y = (e \,\&\, b))$
    N (only formulas can be joined by &, and only variables or constants can go around a =)

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B. Quantifier logic translations: Translate each of the following using the key given. (48 pts).

1. Unless Brad is a tiger without feathers, he hasn't sniffed any birds.
   $(Tb \,\&\, \neg Fb) \vee \neg\exists x(Bx \,\&\, Sbx)$
2. Some hairy tiger is cuter than every animal which sniffed it.
   $\exists x(Tx \,\&\, Hx \,\&\, (x \text{ is cuter than every animal which sniffed it}))$
   $\exists x(Tx \,\&\, Hx \,\&\, \forall y(Syx \rightarrow Cxy))$
3. There's exactly one tiger with feathers, and it's afraid of Alyssa.
   $\exists x((x \text{ is the only tiger with feathers}) \,\&\, Axa)$
   $\exists x(\forall y[(Ty \,\&\, Fy) \leftrightarrow y = x] \,\&\, Axa)$
4. Ella likes any animal she's cuter than better than any animal who's cuter than her.
   $\forall x(Cex \rightarrow (\text{Ella likes } x \text{ better than any animal who's cuter than her}))$
   $\forall x(Cex \rightarrow \forall y(Cye \rightarrow Lexy))$
5. No animal other than Alyssa has sniffed Ella.
   $\neg\exists x(\neg x = a \,\&\, Sxe)$
6. If any animal with feathers also has hair, every other animal is afraid of it.
   $\forall x((Fx \,\&\, Hx) \rightarrow (\text{every other animal is afraid of } x))$
   $\forall x((Fx \,\&\, Hx) \rightarrow \forall y(\neg y = x \rightarrow Ayx))$
7. Only tigers with feathers are cuter than both Brad and Ella.
   $\forall x((Cxb \,\&\, Cxe) \rightarrow (Tx \,\&\, Fx))$
8. Not every bird has sniffed at least one hairy animal who's afraid of every tiger.
   $\neg\forall x(Bx \rightarrow (x \text{ has sniffed at least one hairy animal who's afraid of every tiger}))$
   $\neg\forall x(Bx \rightarrow \exists y[Sxy \,\&\, Hy \,\&\, (y \text{ is afraid of every tiger})])$
   $\neg\forall x(Bx \rightarrow \exists y[Sxy \,\&\, Hy \,\&\, \forall z(Tz \rightarrow Ayz)])$

C. Relating logical concepts: For each of the following, state whether it's true or false, and either explain **in full detail** why it's true or give an example to show that it's false. **Note**: If you give an example which works, you will get full credit. On the other hand, if you give an example that doesn't work, it might be good to have something written about it to help me know what had in mind when I'm assigning any partial credit (also: trying to explain why it works might help you realize it doesn't work). (42 points).

1. If $A \therefore B$ is valid, and $\{B, C\}$ is inconsistent, then $A \therefore C$ is invalid.
   False. For example: A is "No dog is a dog", B is "No cat is a cat", and C is "No man is a man".

2. If all of an argument's premises are tautologies, but its conclusion isn't, the argument is invalid.

True. If the conclusion isn't a tautology, it's possible for it to be false. But the premises must always be true. Therefore, it's possible for the conclusion to be true while the conclusion is false, which means the argument is invalid.

3. If an argument is sound and its conclusion is a tautology, its premises are also tautologies.

False. For example: "Pittsburgh is a city. Therefore, every city is a city."

4. If $A \vee B$ is a tautology, then $\{A, B\}$ is consistent.

False. For example: A is "Fido is a dog" and B is "Fido isn't a dog".

5. If $A \rightarrow B$ is true, then $A \therefore B$ is a valid argument.

False. For example: A is "Pittsburgh is a city" and B is "Ohio is a state".

6. If $\{A, B\}$ is an inconsistent set, then the argument $A \therefore B$ can't be sound.

True. A sound argument has to have true premises, and since it's valid it can't have true premises and a false conclusion, so it's conclusion has to be true as well. So if it were sound, both A and B would be true. But $\{A, B\}$ is inconsistent, so there's no possible world in which both A and B are true, so they're definitely not both true, so the argument can't be sound.

Good luck!