Philosophy 500 — May 17th: More sentential Logic

I. Convert each of the following into symbolic notation, using the given key, and draw a tree diagram to represent it:

A: Art is a grandfather.
B: Blinky is Art's cat.
C: Carol is Art's oldest granddaughter.
D: Disney makes terrible movies.
E: Ernie was on Sesame Street.
F: Floyd is a grandfather.

1. Either Art is a grandfather unless Ernie was on Sesame Street, or Disney makes terrible movies if and only if Blinky is Art's cat.
   $(A \vee E) \vee (D \leftrightarrow B)$
2. If Disney makes terrible movies, then either Ernie was on Sesame Street or Art is a grandfather.
   $D \rightarrow (E \vee A)$
3. Although Ernie was on Sesame Street, Floyd and Art aren't both grandfathers.
   $E \,\&\, \neg(F \,\&\, A)$
4. If Blinky is Art's cat, then Floyd is a grandfather but Art isn't.
   $B \rightarrow (F \,\&\, \neg A)$
5. Either Floyd isn't a grandfather, or Art isn't, but Disney makes terrible movies either way.
   $(\neg F \vee \neg A) \,\&\, D$
6. It's not the case that Disney doesn't make terrible movies, but it is the case that Disney does make terrible movies and Carol is Art's oldest granddaughter.
   $\neg\neg D \,\&\, D \,\&\, C$
7. Disney only makes terrible movies if Blinky is Art's cat, but if Art either is or is not a grandfather, then Disney makes terrible movies.
   $(D \rightarrow B) \,\&\, [(A \vee \neg A) \rightarrow D]$
8. It isn't the case that Floyd is a grandfather or that Ernie was on Sesame Street, but—unless Disney makes terrible movies—Art is a grandfather.
   $\neg(F \vee E) \,\&\, (D \vee A)$
9. Art is a grandfather and either Floyd is one too or Disney makes terrible movies.
   $A \,\&\, (F \vee D)$

II. For each of the following, find out whether it's a proper sentence or gibberish by trying to draw a tree diagram for it. *1, 3, 7, 11, and 13 are proper sentences*

1. $A \,\&\, (B \to C) \,\&\, \neg(A \vee B)$
2. $A \to [A(B \,\&\, C)]$
3. $\neg\neg(A \vee \neg(B \,\&\, \neg D))$
4. $\leftrightarrow (A \,\&\, C) \to B$
5. $[A \,\&\, (B \vee \neg C)] \to (A \neg A)$
6. $A \,\&\, B \,\&\, C \vee D$
7. $[(A \to B) \,\&\, (B \to C)] \to (A \to C)$
8. $A \to (B \,\&\, (C \neg \,\&\, A))$
9. $(A \,\&\, B) \vee (A \,\&\, \neg B) \to A$
10. $(C \,\&\, (\neg A \vee BD))$
11. $C \leftrightarrow \neg(\neg A \,\&\, B)$
12. $A \,\&\, \neg(\vee AB) \,\&\, D$
13. $(A \leftrightarrow B) \,\&\, [(\neg A \,\&\, C) \vee \neg D] \,\&\, E$

Relating logical concepts

For each of the follow, state whether it's true or false. If it's true, explain why. If it's false, give an example which shows that.

1. If $A \leftrightarrow B$ is contingent, so is $A \to B$.

First thoughts: *Remember that $A \leftrightarrow B$ is true if both are true or if both are false, and it's false if one of them is true and one is false. Therefore, its being contingent means that it's possible for A and B to have the same truth value, and it's possible for them to have different truth values. Now, for $A \to B$ to be contingent means that it's possible for A to be true and B false, and also possible for this not to be the case. How are we to connect these two things? Well, if A and B have the same truth value, then it's not the case that A is true and B false. In other words, if $A \leftrightarrow$ is true, either A and B are both true, or A and B are both false, and in either case $A \to B$ comes out false. So it's definitely possible for $A \to B$ to be true if it's possible for $A \leftrightarrow B$ is contingent. The question then is whether we can affirm that $A \to B$ could be false. In other words, does it follow that it's possible for A to be true and B false. The answer is no, all we know is that it's possible for one of them to be true and the other false, but maybe it's possible for A to be false and B true, but not the other way around. This will lead us to the counter-example.*

Answer: *False. For example, if A stands for "Bill is both tall and not tall", and B stands for "Pittsburgh is in the US". It's possible for both to false (if Pittsburgh weren't in the US) and it's possible for A to be false and B true, so $A \leftrightarrow B$ is contingent. But it isn't possible for A to be true and B false, so $A \to B$ is a tautology.*

2. If $\{A, B, C\}$ is an inconsistent set, then $\{A, B, \neg C\}$ is a consistent set.

First thoughts: *That it's an inconsistent set just tells us that A, B, and C can't all be true at the same time. In other words, if we knew two of them to be true we would know the other one is false. But that's all it tells us. We don't know whether the contradiction requires all 3 of them, or whether some subset of them is already contradictory. This leads us to the example.*

Answer: *False. For example, if A is "Bill is both tall and not tall, B is "Ted is a man", and C is "Sandy is tall". Any set containing A would be inconsistent.*

3. If A isn't a proper sentence, neither is $\neg A$.

First thoughts: *We have to ask ourselves whether we could have $\neg A$ be a sentence, even though A isn't. For this, we have to look at the rules of how sentences are built.*

Answer: *True. The only rule that introduces a $\neg$ says that if B is a sentence, so is $\neg B$. Therefore, there is no way to introduce an $\neg$ in front of something that isn't a sentence, so unless A is a proper sentence, $\neg A$ won't be one.*

4. If $A \leftrightarrow B$ is a tautology, so is $\neg A \vee B$.

First thoughts: *That $A \leftrightarrow B$ is a tautology, means that it's impossible to be false, which means that it's impossible for A to be true while B is false. The question is whether $\neg A \vee B$ is a tautology. So we want to know whether it's possible for it to be false. But $\neg A \vee B$ is false if and only if A is true and B is false, so we see the connection.*

Answer: *True. If $A \leftrightarrow B$ is a tautology, then it's impossible for A to be true while B is false. But the only way for $\neg A \vee B$ to be false is if A is true and B is false. Therefore, it's impossible for $\neg A \vee B$ to be false, which makes it a tautology.*

5. If $A \,\&\, B$ is logically equivalent to $A \vee B$, then A is logically equivalent to B.

First thoughts: *Their being logically equivalent means two things. First, that if $A \,\&\, B$ is true, so is $A \vee B$, and, second, that if $A \vee B$ is true, so is $A \,\&\, B$. In other words, first that if both A and B are true, at least one of them is, and second that if at least one of them is true they both are. The first part of this is always true, regardless of what A and B are, so we can disregard it. Instead we'll focus on the fact that if one of them is true they both are. What does this mean?*

Answer: *True. If $A \,\&\, B$ and $A \vee B$ are logically equivalent, it's impossible for one of them to be true while the other is false. In particular, it's impossible for $A \vee B$ to be true while $A \,\&\, B$ is false. In other words, if either A or B is true, then they both are. That means that if A is true, so is B, and that if B is true, so is A. But then it's impossible for one of them to be true while the other is false, so they're logically equivalent.*

6. If $A \vee B$ is contingent, so are A and B.

First thoughts: *That $A \vee B$ is contingent means that it's possible for it to be true (i.e. for at least one of the two to be true) and that it's possible for it to be false (i.e. for both A and B to be false). The second part tells us that A could be false and B could be false. The question is whether A and B are contingent, so what we need to know is whether we can affirm that it's possible for A to be true and possible for B to be true. But all we know is that it's possible for at least one of them to be true, which leads us to a counter-example.*

Answer: *False. For example, if A is "Bill is tall" and B is "Alice is both tall and not tall". $A \vee B$ would be true in worlds in which Bill is tall and false in worlds in which Bill isn't tall, so it's contingent. But B is a contradiction.*

7. If the argument $A, A \vee B \therefore C$ is sound, so is $A \therefore C$

First thoughts: *If the first argument is sound, it means that A and $A \vee B$ together imply C, and that they're both true. We want to know whether A implies C and whether A is true. The second part is easy, since we already know that A is true since both A and $A \vee B$ is true. The question then is whether A would imply C.*

Answer: *Since the first argument is sound, A must be true, so the second argument's premise is true. Now, to see whether it's possible for A to be true while C is false. [Note that we can't just say "C is true since the first argument is sound": the question is not whether C is true, but whether A logically implies C.] If A were true and C false, then $A \vee B$ would also be true, since it says that one out of A or B is true. But then this would mean the first argument is invalid, which isn't the case. So it's not possible, which shows that the second argument is valid. Combined with what we had before, this shows it's sound.*

8. If an argument is valid but not sound, there is a logically possible world in which it would be sound.

First thoughts: *That the argument isn't sound means that it's premises are false. Now the question is whether there is some possible world in which it would be sound. To be sound means to be valid and to have true premises. Being valid or not doesn't depend on the world, since it is a question of what's possible, not what's the case. In other words, this argument would be valid in any world. So really we just want to know whether there is a possible world in which its premises would be true, which leads us to the counterexample.*

Answer: *False. For example: "Bill is both tall and not tall. Therefore, Bill is tall". This is valid, since its premise can't be true, so it's not possible for its premise to be true and its conclusion false. It's not sound, since the premise is false. And there is no logically possible world in which it would be sound, since there is no possible world in which Bill would be both tall and not tall.*