Preimages of trionometric functions

Mr. Neeman. 10B, October 25, 2011.

Practice exercises

#P1. Suppose $f(x) = \cos x$. Find the preimages of $\frac{1}{\sqrt{2}}$.

#P2. Suppose $f(x) = \sin x$. Find any preimages of 0 in the interval $[-4\pi, 0]$.

#P3. Suppose $f(x) = \tan x$. Find the preimages of -2.

Homework

#H1. Suppose $f(x) = \tan x$. Find any preimages of $\sqrt{3}$ in the interval $[-2\pi, 2\pi]$.

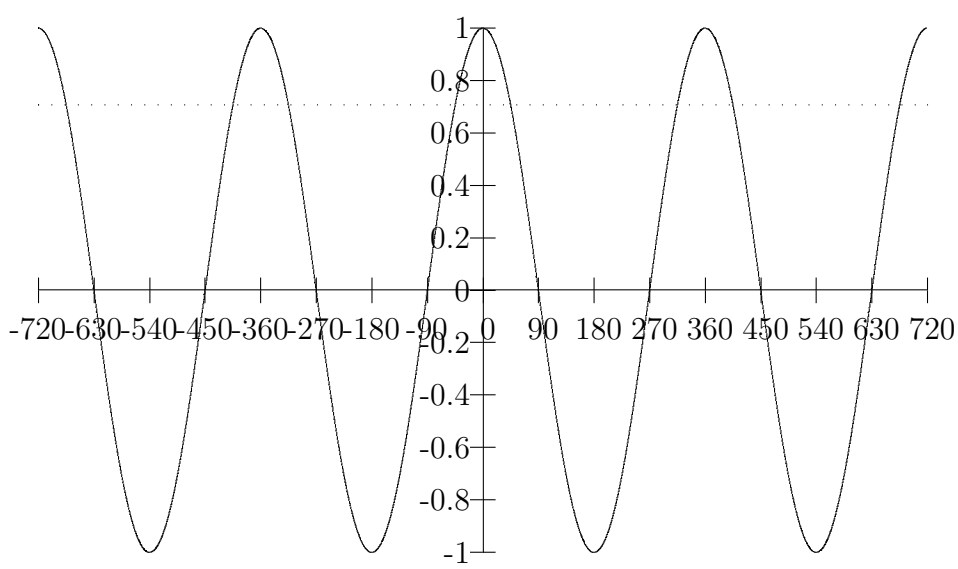
#H2. Suppose $f(x) = \cos x$. Find the preimages of -1.

#H3. Suppose $f(x) = \sin x$. Find the preimages of -0.2588.

Solutions for practice exercises

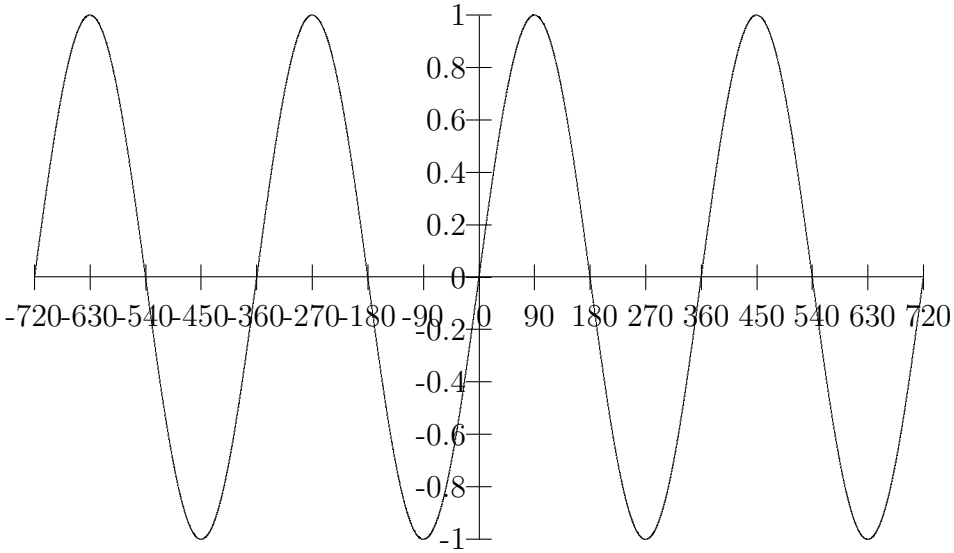
#P1. Suppose $f(x) = \cos x$. Find the preimages of $\frac{1}{\sqrt{2}}$.

$\cos x = \frac{1}{\sqrt{2}}$
To find the first preimage, we take the inverse cosine (or use special triangles):
$x = \cos^{-1} \frac{1}{\sqrt{2}} = 45°$.
To get an idea of the preimages, we look at the graph and draw the line $y = \frac{1}{\sqrt{2}}$.



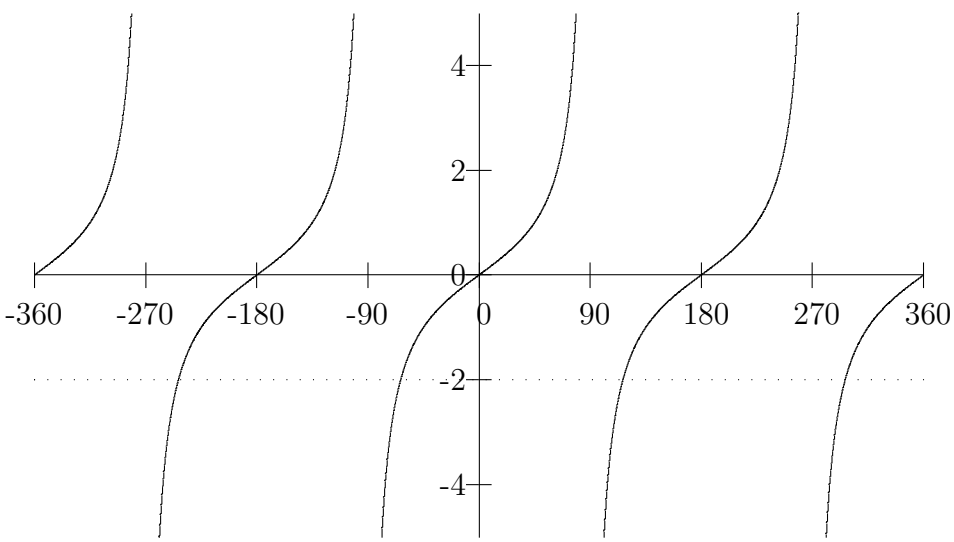
One solution was 45°. We see another one is its reflection around the y axis, which means it's $-45°$. This makes sense since we know $\cos x = \cos(-x)$ since cosine is an even function. Now, these two solutions are repeated every 360°.
So our answer is $x = 45° + 360k°$ and $-45° + 360k°$, with $k \in \mathbb{Z}$.

#P2. Suppose $f(x) = \sin x$. Find any preimages of 0 in the interval $[-4\pi, 0]$.
We can find one preimage by taking the inverse sine of 0, which gives us $x = 0°$. Now, we can find the others by looking at the graph.



We can see there's a solution every 180°. But we're only looking for solutions in the interval $[-4\pi, 0]$. So the solutions are $-720°, -540°, -360°, -180°$, and $0°$.

#P3. Suppose $f(x) = \tan x$. Find the preimages of -2.
We can find a preimage by doing $x = \tan^{-1} -2 \approx -63.4°$.
Looking at the graph, we see that this solution repeats itself every cycle, which is every 180°, since the period of tangent is 180°.



So our answer is $x = -63.4° + 180k°$, with $k \in \mathbb{Z}$.