Power laws

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Power laws

Power laws are just rules we can use to manipulate algebraic expressions involving powers (also called exponents or indices). Suppose throughout that a and b are positive numbers.

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{mn}$
3. $(ab)^n = a^n b^n$
4. $(\frac{a}{b})^n = \frac{a^n}{b^n}$
5. $a^{-n} = \frac{1}{a^n}$
6. $\frac{a^n}{a^m} = a^{n-m}$
7. $a^0 = 1$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Exercise: simplify each of the following

#1. $\frac{9^6}{3^{10}}$

#2. $\frac{a^3}{a^{-n}}$

#3. $\frac{1}{(ab)^3} a^{-2} b^4$

#4. $\frac{(ab)^{-m}}{(\sqrt{b})^{-2m}}$

#5. $\frac{(\frac{a^m}{b^3})}{(\frac{(b^2)^3}{a^{-5}})}$

#6. $(\frac{a}{2})^3 (\frac{2}{b})^{-2}$

#7. $\frac{(a^{\frac{3}{2}})^3}{\sqrt{a}}$

#8. $\sqrt[3]{(ab^2)^6}$

#9. $\frac{4^{\frac{5}{2}}\sqrt{a}}{(2a^{\frac{1}{2}})^3}$

#10. $\frac{(a^2b^3)^{\frac{1}{2}} + a^2b^2}{a^{-1}b}$

#11. $\dfrac{(8a^6b^3)^{-\frac{1}{3}}}{\frac{1}{4}a^{-3}}$

#12. $\dfrac{(b^0)^5}{(\frac{1}{2})^{-2}a^{-2}}$

<u>Solutions</u>

Note: Answers don't have to be exactly the same as here. For example, 2^5 or 32 are both simple, a^{-2} and $\dfrac{1}{a^2}$ are equally simple, etc. However, not anything goes (e.g. $\dfrac{a^3}{a^{-2}}$ is not ok, since a^5 is clearly simpler).

#1. $\dfrac{9^6}{3^{10}} = \dfrac{3^{12}}{3^{10}} = 3^2 = 9$

#2. $\dfrac{a^3}{a^{-n}} = a^{3+n}$

#3. $\dfrac{1}{(ab)^3}a^{-2}b^4 = (ab)^{-3}a^{-2}b^4 = a^{-3}b^{-3}a^{-2}b^4 = a^{-5}b$

#4. $\dfrac{(ab)^{-m}}{(\sqrt{b})^{-2m}} = (ab)^{-m}(\sqrt{b})^{2m} = (ab)^{-m}(b^{\frac{1}{2}})^{2m} = (ab)^{-m}b^m = a^{-m}b^{-m}b^m = a^{-m}$

#5. $\dfrac{(\frac{a^m}{b^3})}{(\frac{(b^2)^3}{a^{-5}})} = \dfrac{a^m a^{-5}}{b^3(b^2)^3} = \dfrac{a^{m-5}}{b^9}$

#6. $(\dfrac{a}{2})^3(\dfrac{2}{b})^{-2} = \dfrac{a^3}{2^3}\dfrac{2^{-2}}{b^{-2}} = \dfrac{a^3b^2}{2^5} = \dfrac{a^3b^2}{32}$

#7. $\dfrac{(a^{\frac{3}{2}})^3}{\sqrt{a}} = \dfrac{a^{\frac{9}{2}}}{a^{\frac{1}{2}}} = a^{\frac{8}{2}} = a^4$

#8. $\sqrt[3]{(ab^2)^6} = (ab^2)^{\frac{6}{3}} = (ab^2)^2 = a^2b^4$

#9. $\dfrac{4^{\frac{5}{2}}\sqrt{a}}{(2a^{\frac{1}{2}})^3} = \dfrac{\sqrt{4}^5 a^{\frac{1}{2}}}{2^3a^{\frac{3}{2}}} = \dfrac{2^5a^{\frac{1}{2}}}{2^3a^{\frac{3}{2}}} = \dfrac{2^2}{a} = \dfrac{4}{a}$

#10. $\dfrac{(a^2b^3)^{\frac{1}{2}} + a^2b^2}{a^{-1}b} = (ab^{\frac{3}{2}} + a^2b^2)(ab^{-1}) = (ab^{\frac{3}{2}}ab^{-1} + a^2b^2ab^{-1}) = a^2b^{\frac{1}{2}} + a^3b$

#11. $\dfrac{(8a^6b^3)^{-\frac{1}{3}}}{\frac{1}{4}a^{-3}} = \dfrac{4a^3}{(8a^6b^3)^{\frac{1}{3}}} = \dfrac{4a^3}{2a^2b} = \dfrac{2a}{b}$

#12. $\dfrac{(b^0)^5}{(\frac{1}{2})^{-2}a^{-2}} = \dfrac{1}{(\frac{1}{2})^{-2}a^{-2}} = (\dfrac{1}{2})^2a^2 = \dfrac{a^2}{4}$