Sets and set operations (solutions)

Mr. Neeman. 10B, November 23, 2011.

#1. Suppose the following:

$\Omega = \{x \in \mathbb{Z} | 2 < x < 10\}$
$A = \{x \in \Omega | x \text{ is even}\}$
$B = \{x \in \Omega | x \geq 5\}$
$C = \{x \in \Omega | x \text{ is divisible by } 4\}$

Before doing the others, it will be good to list the elements of Ω:
$\Omega = \{3, 4, 5, 6, 7, 8, 9\}$
(a) List the elements of A.
$A = \{4, 6, 8\}$
(b) List the elements of B.
$B = \{5, 6, 7, 8, 9\}$
(c) List the elements of C.
$C = \{4, 8\}$
(d) List the elements of A'.
$A' = \{3, 5, 7, 9\}$
(e) List the elements of C'.
$C' = \{3, 5, 6, 7, 9\}$
(f) Is A' a subset of C'?
It is, because all its elements are also in C'.
(g) Is A' a proper subset of C'?
It is, because, in addition, they're not equal (6 is in C' but not in A').
(h) Is B a subset of A?
No. For example, 5 is in B but not in A.
(i) List the elements of $(A \cup B)'$
$A \cup B = \{4, 5, 6, 7, 8, 9\}$
So $(A \cup B)' = \{3\}$
(j) List the elements of $A \cap C$.
$A \cap C = \{4, 8\}$
(k) Is $A \cap C$ a subset of C?
It is.
(l) Is $A \cap C$ a proper subset of C?
No, since the two sets are equal.
(m) List the elements of B'.
$B' = \{3, 4\}$
(n) List the elements of $B' \cup C$.
$B' \cup C = \{3, 4, 8\}$
(o) List the elements of $C' \cap C$.
$C' \cap C = \emptyset$ (it has no elements)

#2. For each of the following, say whether it's true or false. Give an example to support your answer, where appropriate (e.g. if you say it's false that $\mathbb{R} \subset \mathbb{Q}$, an example of why would be a real number which isn't rational, such as π; if you say it's true that $\mathbb{N}$ is a proper subset of $\mathbb{R}$, you can give an example of a real number which isn't a natural number (e.g. π), to show the two sets aren't equal).

(a) $\pi \in \mathbb{Z}$
False, π is not a whole number.
(b) $0 \in \mathbb{Q}$
True, 0 is rational, since it can be written, for example, as $\frac{0}{1}$.
(c) $-7 \in \mathbb{N}$
False, negative numbers are not included in the natural numbers.
(d) $\mathbb{N} \subset \mathbb{Q}$
True, every natural number n is also a whole number, so if you write it as $\frac{n}{1}$ you see it's also rational.
(e) $\mathbb{Q} \cup \mathbb{Z} = \mathbb{Q}$.
True. Since all integers are rational, there are no elements in $\mathbb{Z}$ which aren't also in $\mathbb{Q}$. So when you take the union it doesn't add any new elements to $\mathbb{Q}$.
(f) $\sqrt{9} \in \mathbb{Q}$
True, since $\sqrt{9} = 3$.
(g) $\frac{\sqrt{3}}{5} \in \mathbb{Q}$
False, $\sqrt{3}$ can't be written as a fraction of two whole numbers, so neither can $\frac{\sqrt{3}}{5}$.