Cofunctions and reciprocal functions, degrees and radians

Mr. Neeman. 10B, October 6, 2011.

Reciprocal functions

For each of the trigonometric function, there is a reciprocal function:

Cosecant: $\csc\alpha = \frac{1}{\sin\alpha}$
Secant: $\sec\alpha = \frac{1}{\cos\alpha}$
Cotangent: $\cot\alpha = \frac{1}{\tan\alpha}$

Notice that, since $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$.

Cofunctions

By examining a right-angled triangle, we can see that:
$\cos\alpha = \sin(90^\circ - \alpha)$, and
$\cos(90^\circ - \alpha) = \sin A$
When such a relationship hold, the two functions (sine and cosine) are called each other's cofunctions. And this is reflected in their names ('cosine' is the cofunction of 'sine').

Likewise, cosecant is the cofunction of secant:
$\csc\alpha = \sec(90^\circ - \alpha)$, and
$\csc(90^\circ - \alpha) = \sec A$

And cotangent is the cofunction of tangent:
$\cot\alpha = \tan(90^\circ - \alpha)$, and
$\cot(90^\circ - \alpha) = \tan A$

Radians and degrees

A radian is defined as that angle such that, in a circle of radius 1, a sector whose angle is 1 radian will have an arc of length 1. Since the whole circle's circumference is 2π, that means there are 2π radians in the circle, so that $2\pi = 360^\circ$. When there is no degree symbol, it's implied that the angle is in radians.

1 radian $= \frac{180^\circ}{\pi}$
$180^\circ = \pi$ radians

Exercises

#1. Simplify each of the following:
(a) $\csc\alpha\tan\alpha$
(b) $\cot\alpha\tan\alpha$
(c) $\frac{\sec\alpha}{\tan\alpha}$
(d) $\frac{\csc\alpha}{\sec\alpha\cot\alpha}$

#2. Convert each of the following from degrees to radians:

(a) $90°$
(b) $30°$
(c) $270°$
(d) $120°$

#3. Convert each of the following from radians to degrees:

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $\frac{5\pi}{4}$
(d) $\frac{4\pi}{3}$

#4. Calculate each of the following:

(a) $\sin\frac{\pi}{6}$
(b) $\cos\frac{\pi}{4}$
(c) $\tan\frac{\pi}{3}$
(d) $\cot\frac{\pi}{6}$