Proving trigonometric identities

Mr. Neeman. 10B, November 8, 2011.

We can use the trigonometric identities we've encountered already to prove other ones. This will typically also involve normal algebraic techniques such as working with fractions, factorizing, and multiplying out.

E.g. #1. Prove that $\dfrac{\tan^2 x}{\sec x} - \sec x = -\cos x$

$$\frac{\tan^2 x}{\sec x} - \sec x$$
$$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x}} - \frac{1}{\cos x}.$$
$$= \frac{\sin^2 x}{\cos x} - \frac{1}{\cos x}$$
$$= \frac{\sin^2 x - 1}{\cos x}$$
$$= \frac{-\cos^2 x}{\cos x}$$
$$= -\cos x$$

E.g. #2. Prove that $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = 2\sec^2 x$.

$$\frac{1-\sin x}{1-\sin^2 x} + \frac{1+\sin x}{1-\sin^2 x}$$
$$= \frac{1-\sin x + 1 + \sin x}{1-\sin^2 x}$$
$$= \frac{2}{\cos^2 x}$$
$$= 2\sec^2 x$$

E.g. #3. Prove that $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \sec x \csc x$.

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^x x}{\sin x \cos x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$
$$= \frac{1}{\sin x \cos x}$$
$$= \frac{1}{\sec x \csc x}$$

E.g. #4. Prove that $\csc x - \sin x = \cot x \cos x$.

$\csc x - \sin x$

$= \dfrac{1}{\sin x} - \sin x$

$= \dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{1 - \sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x}$

$= \cos x \dfrac{\cos x}{\sin x}$

$= \cos x \cot x$

E.g. #5. Prove that $(\sec^2 x - 1)(\csc^2 - 1) = 1$.

$(\sec^2 x - 1)(\csc^2 - 1)$

$= \left(\dfrac{1}{\cos^2 x} - 1\right)\left(\dfrac{1}{\sin^2 x} - 1\right)$

$= \left(\dfrac{1 - \cos^2 x}{\cos^2 x}\right)\left(\dfrac{1 - \sin^2 x}{\sin^2 x}\right)$

$= \left(\dfrac{1 - \cos^2 x}{\cos^2 x}\right)\left(\dfrac{1 - \sin^2 x}{\sin^2 x}\right)$

$= \left(\dfrac{\sin^2 x}{\cos^2 x}\right)\left(\dfrac{\cos^2 x}{\sin^2 x}\right)$

$= 1$

E.g. #6. Prove that $(\tan x + \cot x)\tan x = \sec^2 x$

$(\tan x + \cot x)\tan x$

$= \tan^2 x + \cot x \tan x$

$= \tan^2 x + 1$

$= \sec^2 x$

E.g. #7. Prove that $(\cot x \sin x - \sin x)(\cos x + \sin x) = 2\cos^2 x - 1$

$(\cot x \sin x - \sin x)(\cos x + \sin x)$

$= \left(\dfrac{\cos x}{\sin x}\sin x - \sin x\right)(\cos x + \sin x)$

$= (\cos x - \sin x)(\cos x + \sin x)$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= 2\cos^2 x - 1$