BLUE VALLEY SCHOOL

Mathematics Department	1st bimester, FS 2011
Mr. Neeman	Total points possible: 20
Quiz #4, 10B	Solutions
Topic: Logarithms	Estimated time: 40 minutes

Instructions: Answer all the questions, showing your work neatly. Remember to check your answers afterwards. Leave your answers exact (don't approximate as decimals).

#1. Express each of the following as a single logarithm:

(a) $\log_5 8 - \log_5(y^2)$ (1 pt)

$= \log_5 \dfrac{8}{y^2}$

(b) $2\log_2(x^2) + \log_2(xy)$ (2 pts)

$= \log_2((x^2)^2) + \log_2(xy)$

$= \log_2(x^4) + \log_2(xy)$

$= \log_2(x^5 y)$

(c) $\log_3 5 + \log_3(xy^{-1}) - \log_3 y$ (2 pts)

$= \log_3(5xy^{-1}) - \log_3 y$

$= \log_3 \dfrac{5xy^{-1}}{y}$

$= \log_3 \dfrac{5x}{y^2}$

#2. Express each of the following in terms of $\log_5 x$ and $\log_5 y$.

(a) $\log_5(3xy)$ (1 pt)

$= \log_5 3 + \log_5(xy)$

$= \log_5 3 + \log_5 x + \log_5 y$

(b) $\log_5 \dfrac{x^2}{y^3}$ (2 pts)

$= \log_5(x^2) - \log_5(y^3)$

$= 2\log_5 x - 3\log_5 y$

(c) $\log_5 \sqrt[3]{x} - \log_5(xy^2)$ (2 pts)

$= \dfrac{1}{3}\log_5 x - \log_5(xy^2)$

$= \dfrac{1}{3}\log_5 x - \log_5 x - \log_5(y^2)$

$= -\dfrac{2}{3}\log_5 x - \log_5(y^2)$

$= -\dfrac{2}{3}\log_5 x - 2\log_5 y$

(d) $\log_5 x - \log_{25} x$ (2 pts)

$= \log_5 x - \dfrac{\log_5 x}{\log_5 25}$

$= \log_5 x - \dfrac{\log_5 x}{\log_5(5^2)}$

$= \log_5 x - \dfrac{\log_5 x}{2}$

$= \dfrac{1}{2}\log_5 x$

#3. Solve $\log_2 x + \log_2(x-3) = 3$ (4 pts)

$\log_2(x(x-3)) = 3$

$2^{\log_2(x(x-3))} = 2^3$

$x(x-3) = 8$

$x^2 - 3x = 8$

We can now complete the square (or use the quadratic formula) to solve the equation.

$x^2 - 3x + \dfrac{9}{4} = 8 + \dfrac{9}{4}$

$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{41}{4}$

$x - \dfrac{3}{2} = \pm\dfrac{\sqrt{41}}{2}$

$x = \dfrac{3}{2} \pm \dfrac{\sqrt{41}}{2}$

Now, $\sqrt{41}$ is between 6 and 7. So $\dfrac{\sqrt{41}}{2}$ is between 3 and 3.5. This means that the larger root (when we take the plus from the plus or minus) will be bigger than 3, so that x and $x-3$ will both be positive, so it is a solution. On the other hand, if we take the minus, then x will be negative, since $\sqrt{41}$ is bigger than 3, so that $\dfrac{\sqrt{41}}{2} > \dfrac{3}{2}$. So this solution doesn't work, since it makes the logarithms in our equation be undefined.

Therefore, $x = \dfrac{3}{2} + \dfrac{\sqrt{41}}{2}$

#4. Solve $\log_3(x^2) = 3$ (3 pts)
$3^{\log_3(x^2)} = 3^3$
$x^2 = 27$
$x = \pm\sqrt{27}$
Now, with both $x = \sqrt{27}$ and with $x = -\sqrt{27}$ we get x^2 being positive. So they're both solutions. The reason I gave an extra $\frac{1}{2}$ for finding both is that there's a way which is wrong but for reasons which are not easy to spot. One could also try:
$2\log_3 x = 3$
$\log_3 x = \frac{3}{2}$
$x = 3^{\frac{3}{2}}$
$x = \sqrt{27}$
The catch is that the logarithm law which we use in the first step only works if x is positive, whereas from the original equation all we know is that x isn't zero, there's nothing to rule out its being negative. So technically, this method isn't allowed, which is also why it missed out on one of the solutions.

#5. Express $\log_{\frac{1}{2}}(x^2)$ in terms of $\log_4 x$. (1 pt)
We can do this in two ways:
$\log_{\frac{1}{2}}(x^2)$
$= 2\log_{\frac{1}{2}} x$
$= \frac{2\log_4 x}{\log_4 \frac{1}{2}}$
$= \frac{2\log_4 x}{\log_4(4^{-\frac{1}{2}})}$
$= \frac{2\log_4 x}{-\frac{1}{2}}$
$= -4\log_4 x$
Or we can do the change of base first:
$\log_{\frac{1}{2}}(x^2)$
$= \frac{\log_4(x^2)}{\log_4 \frac{1}{2}}$
$= \frac{\log_4(x^2)}{\log_4(4^{-\frac{1}{2}})}$
$= \frac{\log_4(x^2)}{-\frac{1}{2}}$
$= -2\log_4(x^2)$
$= -4\log_4 x$