BLUE VALLEY SCHOOL

Mathematics Department	1st bimester, FS 2011
Mr. Neeman	Total points possible: 37
Midterm exam #1, 10B	Solutions
Topic: Quadratic and exponential functions	Estimated time: 80 minutes

Instructions: Answer all the questions. Remember to check your answers afterwards.

Section A: Multiple choice, circle the right answer (1 pt each).

1. The function $f(x) = 3x^2 - 1$ is

A. Neither injective nor surjective

2. Which of the following is NOT true? Assume $a, b > 0$.

D. $a^n b^m = (a + b)^{n+m}$

3. Which of the following is NOT an exponential function?

C. $f(x) = (-2)^x$

4. What are the coordinates of the vertex of $f(x) = x^2 - 4x + 7$?

B. $(2, 3)$

5. The function $f(x) = -x^2 + 1$ is increasing on

B. $]-\infty, 0]$

6. Under the function $f(x) = -x^2 - 5$, how many preimages does -2 have?

A. 0

Section B: Detailed exercises, answer each of the following questions, showing clearly any working.

1. Consider the function $f(x) = x^2 + 2x - 3$.

(a) Find the coordinates of its vertex. (2 pt)
$f(x) = (x + 1)^2 - 4$.
So the vertex is at $(-1, -4)$.

(b) Find its intersection with the y axis. (1 pt)

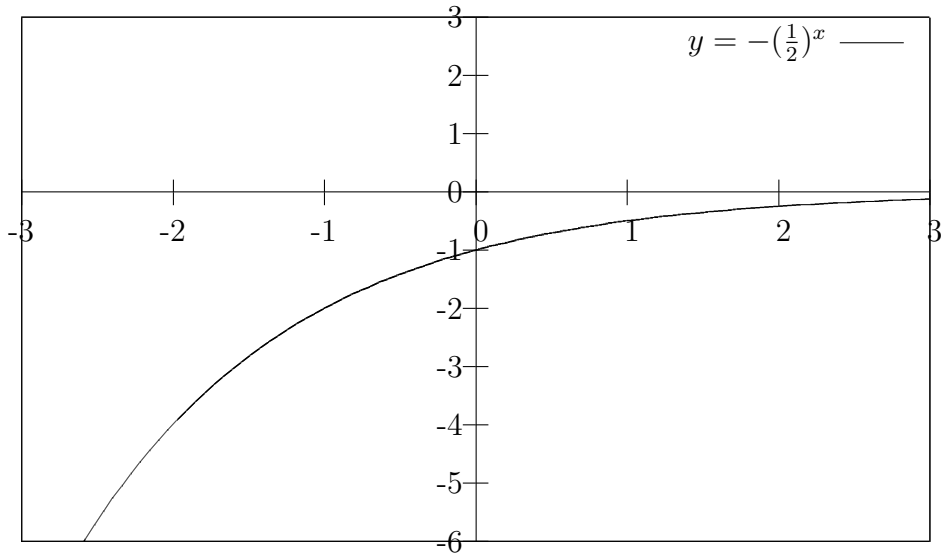
$f(0) = -3$.

(c) Find its intersections, if there are any, with the x axis. (1 pt)

We set $f(x) = 0$: $(x + 1)^2 - 4 = 0$. Therefore, $x + 1 = \pm 2$, so $x = -3$ or $x = 1$.
Therefore the intersections with the x axis are at -3 and 1.

2. Consider the function $f(x) = -(\frac{1}{2})^x$

(a) Sketch the function's graph. (2 pt)



(b) Is the function injective? (1 pt)

Yes.

(c) What is the function's concavity? (1 pt)

Concave down.

(d) Find the image of 4. (1 pt)

$f(4) = -(\frac{1}{2})^4 = -\frac{1}{16}$.

(e) Find the preimage of $-\sqrt{8}$. (1 pt)

$-(\frac{1}{2})^x = -\sqrt{8}$
$(\frac{1}{2})^x = \sqrt{8}$
$(\frac{1}{2})^x = 2^{\frac{3}{2}}$
$\frac{1}{2^x} = 2^{\frac{3}{2}}$
$2^{-x} = 2^{\frac{3}{2}}$
$-x = \frac{3}{2}$
$x = -\frac{3}{2}$

(f) Find the function's range. (1 pt)

$]-\infty, 0[$.

3. Solve the following exponential equations:

(a) $(\frac{1}{2})^{-x} = 4$ (1 pt)
$\frac{1}{2^{-x}} = 4$
$2^x = 4$
$2^x = 2^2$
$x = 2$

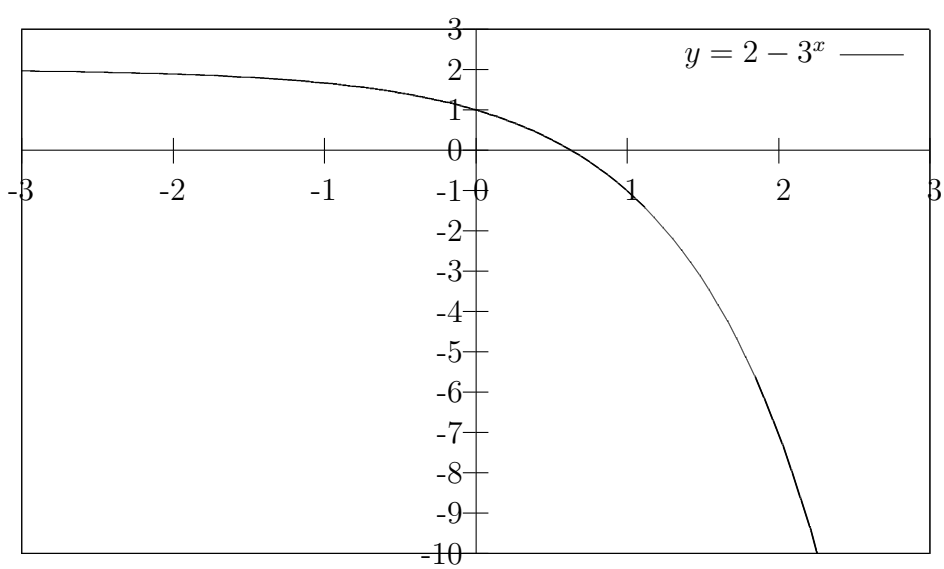
(b) $3^{2x+1} = 2 - 5(3^x)$ (5 pts)
$3(3^{2x}) + 5(3^x) - 2 = 0$
$3(3^x)^2 + 5(3^x) - 2 = 0$
Substitute $y = 3^x$
$3y^2 + 5y - 2 = 0$
$(3y - 1)(y + 2) = 0$
Therefore, $y = \frac{1}{3}$ or $y = -2$.
If $y = \frac{1}{3}$, then $3^x = \frac{1}{3}$, which means $x = 1$.
If $y = -2$, then $3^x = -2$, which has no solutions.
Therefore, $x = -1$.

(c) $4^{x^2-3} = \sqrt{2}^{2x}$ (3 pts)
$2^{2x^2-6} = 2^x$
$2x^2 - 6 = x$
$2x^2 - x - 6 = 0$
$(2x + 3)(x - 2) = 0$
Therefore, $x = -\frac{3}{2}$ or $x = 2$.

5. Simplify $\dfrac{(\sqrt[3]{ab^2})^4}{a^2b^{-1}}$. (1 pt)

$$\frac{(\sqrt[3]{ab^2})^4}{a^2b^{-1}} = \frac{(ab^2)^{\frac{4}{3}}}{a^2b^{-1}} = \frac{a^{\frac{4}{3}}b^{\frac{8}{3}}}{a^2b^{-1}} = a^{-\frac{2}{3}}b^{\frac{11}{3}}$$

6. Sketch the function $f(x) = 2 - 3^x$. (2 pts)



4. Consider the function $f(x) = -2x^2 + 8x - 4$

(a) Find the coordinates of its vertex. (2 pts)

$f(x) = -2(x^2 + 4x + 2) = -2((x-2)^2 - 2) = -2(x+2)^2 + 4$
Therefore, the vertex is at $(2, 4)$.

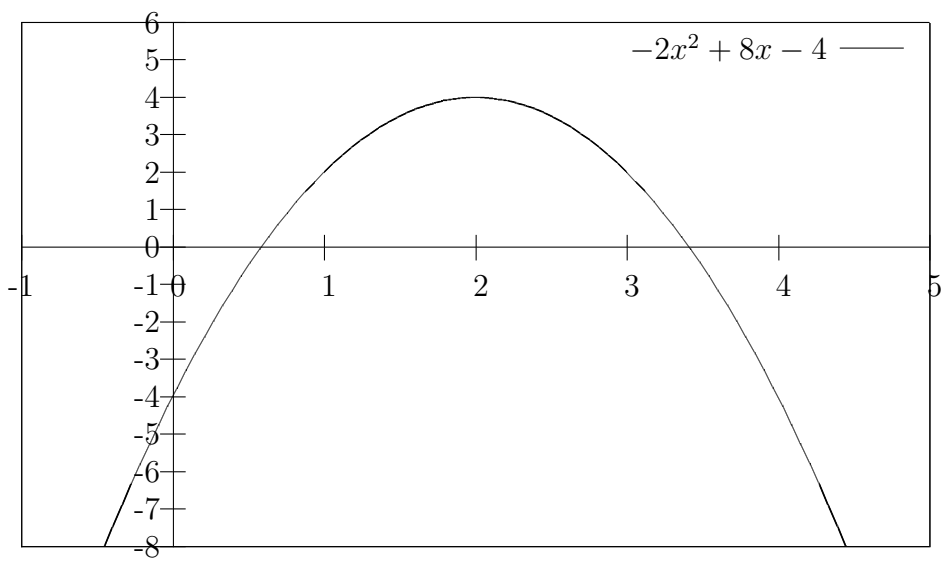
(b) Find its intersection with the y axis. (1 pt)

$f(0) = -4$

(c) Find its intersections, if there are any, with the x axis. (1 pt) $-2(x-2)^2 + 4 = 0$

$(x-2)^2 = 2$
$x = 2 \pm \sqrt{2}$
Therefore, the intersections with the x axis are $2 - \sqrt{2}$ and $2 + \sqrt{2}$.

(d) Sketch the function's graph, labeling clearly the features found in (a)-(c). (2 pt)



(and everything needs to be labeled)

(e) Find the function's range. (1 pt)

$]-\infty, 4]$.

(f) Is the function injective? (1 pt)

No.