Quadratic function review (solutions)

Mr. Neeman, Aug. 24.

For each of the following functions, find (a) the equation of the axis of symmetry, coordinates of the vertex, (b) the coordinates of the vertex, (c) all intersections with the axes, (d) sketch the graph, (e) its concavity, (f) its range, (g) whether it's injective, surjective, both, or neither, (h) the indicated image, (i) the indicated preimages if they exist, (j) where it's increasing and where it's decreasing.

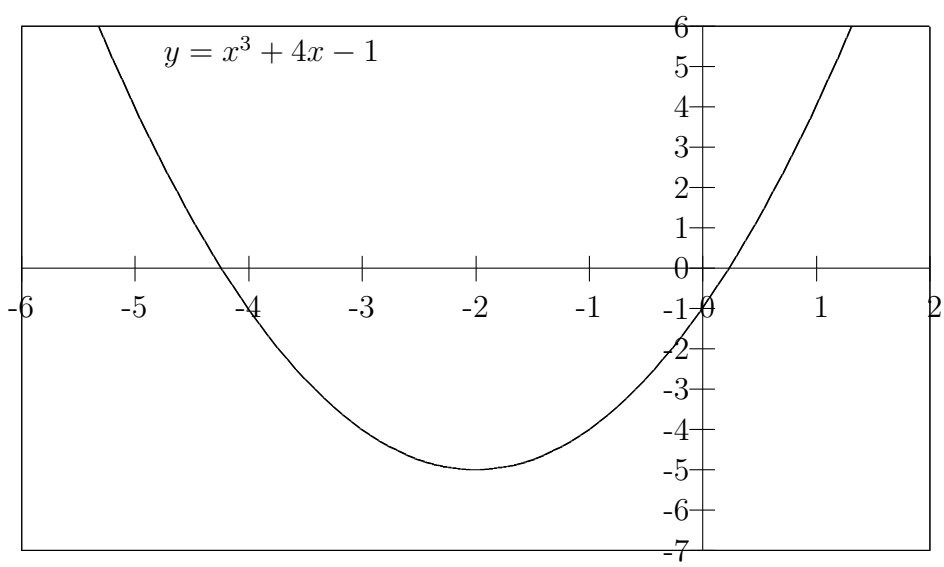
#1. $f(x) = x^2 + 4x - 1$, image of 3, preimage of -5.

(a) Completing the square, $f(x) = (x+2)^2 - 5$. This means the axis of symmetry has equation $y = -2$.

(b) $(-2, -5)$

(c) Intersection with y axis is at $y = f(0) = -1$.
For intersections with x axis, we set $f(x) = 0$, so $(x+2)^2 - 5 = 0$. This means $x = -2 \pm \sqrt{5}$, so the intersections are at $x = -2 - \sqrt{5}$ and $x = -2 + \sqrt{5}$.

(d) The sketch is below. You should also include the intersections found above and the coordinates of the vertex and labels for the axes and the curve.



(e) Concave up

(f) Range is $[-5, \infty[$

(g) Neither injective nor surjective.

(h) $f(3) = 20$

(i) $(x+2)^2 - 5 = -5$. Therefore, $(x+2)^2 = 0$, so $x = -2$.

(j) It's increasing on $[-2, \infty[$ and decreasing on $]-\infty, -2]$ (the vertex is included, because the question doesn't say 'strictly increasing'.

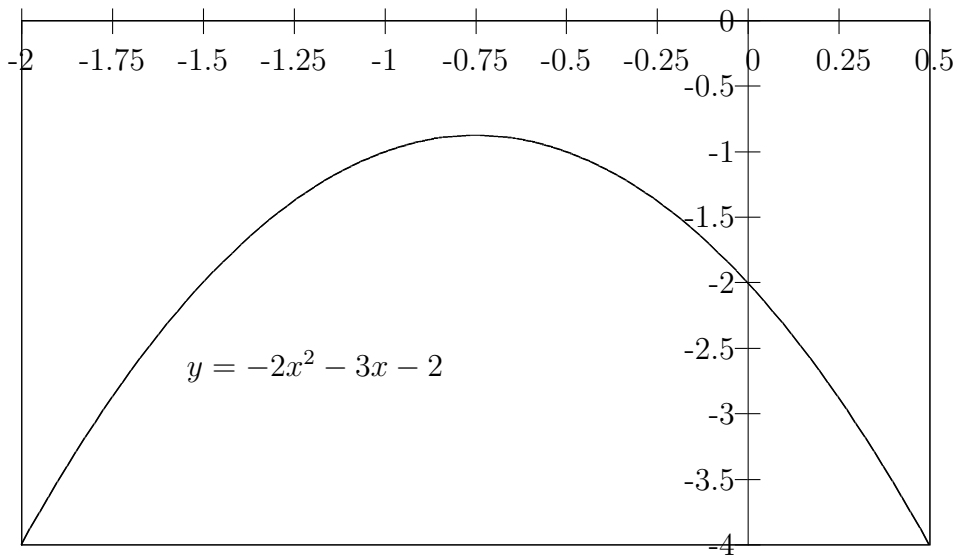
#2. $f(x) = -2x^2 - 3x - 2$, image of $\frac{2}{3}$, preimage of 2.

(a) Completing the square, $f(x) = -2(x + \frac{3}{4})^2 - \frac{7}{8}$. This means the axis of symmetry has equation $y = -\frac{3}{4}$.

(b) $(-\frac{3}{4}, -\frac{7}{8})$

(c) Intersection with y axis is at $y = f(0) = -5$.
For intersections with x axis, we set $f(x) = 0$, so $-2(x + \frac{3}{4})^2 - \frac{7}{8} = 0$. This means $2(x + \frac{3}{4})^2 + \frac{7}{8} = 0$, which has no solutions.

(d) The sketch is below. You should also include the intersection found above and the coordinates of the vertex and labels for the axes and the curve.



(e) Concave down

(f) Range is $]-\infty, -\frac{7}{8}]$

(g) Neither injective nor surjective.

(h) $f(\frac{2}{3}) = -\frac{44}{9}$

(i) $-2(x + \frac{3}{4})^2 - \frac{7}{8} = 2$. Therefore, $2(x + \frac{3}{4})^2 = -2 - \frac{7}{8}$, which has no solutions. So there are no preimages of 2.

(j) It's increasing on $]-\infty, -\frac{3}{4}]$ and decreasing on $[-\frac{3}{4}, \infty[$.

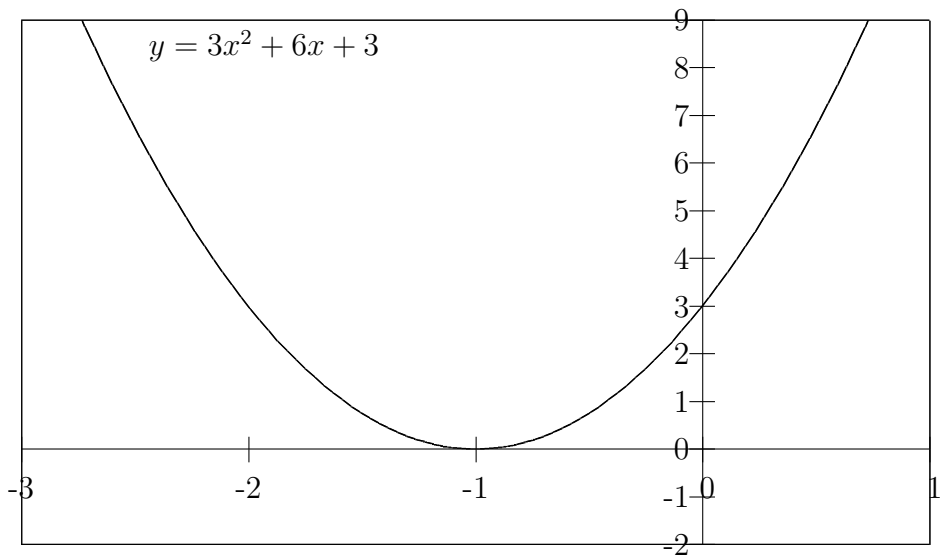
#3. $f(x) = 3x^2 + 6x + 3$, image of -2, preimage of 4.

(a) Completing the square, $f(x) = 3(x+1)^2$. This means the axis of symmetry has equation $y = -1$.

(b) $(-1, 0)$

(c) Intersection with y axis is at $y = f(0) = 3$.
For intersections with x axis, we set $f(x) = 0$, so $3(x+1)^2 = 0$, so $x = -1$.

(d) The sketch is below. You should also include the intersection found above and the coordinates of the vertex and labels for the axes and the curve.



(e) Concave up.

(f) Range is $[0, \infty[$.

(g) Neither injective nor surjective.

(h) $f(-2) = 3$.

(i) $3(x+1)^2 = 4$. Therefore, there are two preimages: $x = -1 - \frac{2}{\sqrt{3}}$, and $x = -1 + \frac{2}{\sqrt{3}}$.

(j) It's increasing on $[-1, \infty[$ and decreasing on $]-\infty, -1]$.