Selected homework solutions: change of base and logarithmic equations

Mr. Neeman, 10B, Sept. 9, 2011.

Homework (for Monday, Sept. 12)

#H3. Express $\log_3(x^4) - \log_9(y^2)$ as a single logarithm.

$\log_3(x^4) - \log_9(y^2)$

$\log_3(x^4) - \dfrac{\log_3 y^2}{\log_3 9}$

$\log_3(x^4) - \dfrac{\log_3 y^2}{2}$

$\log_3(x^4) - \log_3 y$

$\log_3 \dfrac{x^4}{y}$

#H7. Solve $\log_2 x + 2 = \log_2(2x+4)$.

$2 = \log_2(2x+4) - \log_2 x$

$2 = \log_2 \dfrac{2x+4}{x}$

$4 = \dfrac{2x+4}{x}$

$4x = 2x + 4$

$2x = 4$

$x = 2$

#H10. Solve $\log_2(x-2) = \log_4(x^3-8)$.

$\log_2(x-2) = \log_4(x^3-8)$

$\log_2(x-2) = \dfrac{\log_2(x^3-8)}{\log_2 4}$

$\log_2(x-2) = \dfrac{\log_2(x^3-8)}{2}$

$2\log_2(x-2) = \log_2(x^3-8)$

$\log_2(x-2)^2 = \log_2(x^3-8)$

$(x-2)^2 = (x^3-8)$

$(x-2)^2 = (x-2)(x^2+2x+4)$

So $x-2$ would be a solution. However, this can't be a solution, since it would make the logarithm undefined. So $x-2 \neq 0$, so we can divide both sides by it:

$x - 2 = x^2 + 2x + 4$

$0 = x^2 + x + 6$

This has no solutions, since the discriminant is positive. Therefore, there are no solutions.