Logarithmic functions: Part 2

Mr. Neeman. 10B, August 30th, 2011

Recall from before:

(L1)	$\log_a(a^x) = x$
(L2)	$a^{\log_a x} = x$
(L3)	$y = \log_a x$ if and only if $a^y = x$

We can sometimes use these properties to find images and preimages of logarithmic functions. Genereally speaking, this will only be possible in special cases where we're dealing with powers of our base.

Suppose $f(x) = \log_a x$. To find the image of x, we want to find the value y such that $y = \log_a x$. But, using property (L3), this means $x = a^y$. So finding the image is like solving this exponential equation, which we already studied how to do in some cases.

E.g. #1. Suppose $f(x) = \log_2 x$. Find:

(a) The image of 32.

$y = \log_2 32$

Using (L3), this means $2^y = 32$, so that $2^y = 2^5$, and $y = 5$.

(b) The preimage of $\frac{1}{4}$.

$\frac{1}{4} = \log_2 x$

Using (L3), this means $x = 2^{\frac{1}{4}} = \sqrt[4]{2}$.

(c) The image of $\frac{1}{4}$.

$y = \log_2 \frac{1}{4}$

Using (L3), this means $2^y = \frac{1}{4}$, so that $2^y = 2^{-2}$, and $y = -2$.

(d) The preimage of 0.

$0 = \log_2 x$

Using (L3), this means $x = 2^0 = 1$.

<u>Exercises</u>

#P1. Consider the function $f(x) = \log_3 x$.

Find (a) the image of $\sqrt{27}$, (b) the preimage of -2, (c) the image of $\frac{1}{\sqrt{3}}$, (d) the preimage of $\frac{2}{3}$.

#P2. Solve $\log_5(3x - 1) = 2$.

#P3. Consider the function $f(x) = \log_4 x$.

Find (a) the image of $\frac{1}{8}$, (b) the preimage of -1, (c) the image of 1.

#P4. Solve $6^{5x} = 23$.

#P5. Consider the function $f(x) = \log_{\frac{1}{2}} x$.

Find (a) the image of 8, (b) the preimage of $\frac{1}{2}$, (c) the image of $\sqrt{2}$.

#P6. Solve $\log_{\frac{1}{3}}\left(\frac{1}{x}\right) = -2$.

#P7. Solve $\left(\frac{1}{5}\right)^{3x+2} = 7$.

#P8. Consider the function $f(x) = \log_{10} x$.
Find (a) the image of $\sqrt{100000}$, (b) the preimage of -4, (c) the image of 0.01.

#P9. Solve $3 + \log_{\frac{1}{7}}(x^2) = 2$

#P10. Solve $3(5^{2x}) + 4 = 12$

<u>Solutions</u>

#P1. Consider the function $f(x) = \log_3 x$. Find:
(a) the image of $\sqrt{27}$
$\log_3 \sqrt{27} = \log_3(3^{\frac{3}{2}}) = \frac{3}{2}$
(b) the preimage of -2
$-2 = \log_3 x$
$3^{-2} = x$
$x = \frac{1}{9}$
(c) the image of $\frac{1}{\sqrt{3}}$
$\log_3\left(\frac{1}{\sqrt{3}}\right) = \log_3 3^{-\frac{1}{2}} = -\frac{1}{2}$.
(d) the preimage of $\frac{2}{3}$.
$\frac{2}{3} = \log_3 x$
$3^{\frac{2}{3}} = x$
$x = \sqrt[3]{9}$

#P2. Solve $\log_5(3x-1) = 2$.
$3x - 1 = 5^2$
$3x = 25 + 1$
$x = \frac{26}{3}$

#P3. Consider the function $f(x) = \log_4 x$. Find
(a) the image of $\frac{1}{8}$
$\log_4 \frac{1}{8} = \log_4(8^{-\frac{3}{2}}) = -\frac{3}{2}$
(b) the preimage of -1
$-1 = \log_4 x$
$4^{-1} = x$
$x = \frac{1}{4}$
(c) the image of 1.
$\log_4 1 = \log_4(4^0) = 0$

#P4. Solve $6^{5x} = 23$.
$5x = \log_6 23$
$x = \dfrac{\log_6 23}{5}$
#P5. Consider the function $f(x) = \log_{\frac{1}{2}} x$. Find
(a) the image of 8
$\log_{\frac{1}{2}} 8 = \log_{\frac{1}{2}} \left(\left(\dfrac{1}{2} \right)^{-3} \right) = -3$
(b) the preimage of $\dfrac{1}{2}$
$\dfrac{1}{2} = \log_{\frac{1}{2}} x$
$\left(\dfrac{1}{2} \right)^{\frac{1}{2}} = x$
$x = \dfrac{1}{\sqrt{2}}$
(c) the image of $\sqrt{2}$.
$\log_{\frac{1}{2}} \sqrt{2} = \log_{\frac{1}{2}} \left(\left(\dfrac{1}{2} \right)^{-\frac{1}{2}} \right) = -\dfrac{1}{2}$

#P6. Solve $\log_{\frac{1}{3}} \left(\dfrac{1}{x} \right) = -2$.
$\dfrac{1}{x} = \left(\dfrac{1}{3} \right)^{-2}$
$\dfrac{1}{x} = 9$
$x = \dfrac{1}{9}$

#P7. Solve $\left(\dfrac{1}{5} \right)^{3x+2} = 7$.
$3x + 2 = \log_{\frac{1}{5}} 7$
$x = \dfrac{\log_{\frac{1}{5}} 7 - 2}{3}$
#P8. Consider the function $f(x) = \log_{10} x$. Find
(a) the image of $\sqrt{100000}$
$\log_{10} \sqrt{100000} = \log_{10}(10^{\frac{5}{2}}) = \dfrac{5}{2}$
(b) the preimage of -4
$-4 = \log_{10} x$
$10^{-4} = x$
$x = 0.0001$
(c) the image of 0.01.
$\log_{10} 0.01 = \log_{10}(10^{-2}) = -2$

#P9. Solve $3 + \log_{\frac{1}{7}}(x^2) = 2$
$\log_{\frac{1}{7}}(x^2) = -1$
$x^2 = \left(\frac{1}{7}\right)^{-1}$
$x^2 = 7$
$x = \pm 7$

#P10. Solve $3(5^{2x}) + 4 = 12$
$3(5^{2x}) = 8$
$5^{2x} = \frac{8}{3}$
$2x = \log_5 \frac{8}{3}$
$x = \frac{\log_5 \frac{8}{3}}{2}$

<u>Homework (due Thursday, Sept 1st))</u>

Note: the first 5 questions are taken from the homework on the Aug. 28 hand-out.

#H1. Solve: $\log_6(3x) = 10$

#H2. Solve: $7^{2-5x} + 2 = 6$

#H3. Solve: $\log_{1.3}(4x - 7) + 2 = 7$

#H4. Solve: $2^{x^2-1} = 5$

#H5. Solve: $0.7^{3x} + 10 = 23$

#H6. Consider the function $f(x) = \log_3 x$. Find (a) the image of $\frac{1}{9}$, (b) the preimage of 0, (c) the image of $\sqrt[5]{9}$.

#H7. Consider the function $f(x) = \log_{\frac{1}{5}} x$. Find (a) the image of 25, (b) the preimage of $-\frac{1}{2}$, the image of $\sqrt{\frac{1}{5}}$, (d) the preimage of 0.

#H8. Consider the function $f(x) = \log_8 x$. Find (a) the image of 2, (b) the preimage of $-\frac{1}{2}$, (c) the image of $\frac{1}{64}$, (d) the preimage of -1.