Sets and set operations

Mr. Neeman. 10B, November 18, 2011.

Notation and basic concepts

We have already worked with sets before. Recall the following:

$\mathbb{R}$ is the set of real numbers.
$\mathbb{Q}$ is the set of rational numbers.
$\mathbb{I}$ is the set of irrational numbers.
$\mathbb{Z}$ is the set of integers (whole numbers).
$\mathbb{N}$ is the set of natural numbers: 0, 1, 2, 3, ...
$\mathbb{N}^*$ is the set of positive integers: 1, 2, 3, ...

Usually, we will be considering various subsets of a **universal set**. We will denote the universal set with the letter Ω. (Often it's denoted with a "U", but we won't do this to avoid confusion with the symbol for union: "$\cup$".) Note, however, that the universal set will not be the same for every exercise we do.
If a universal set isn't given, we can still do some set operations (union and intersection), but we can't take complements of sets.

We may specify a set by listing its members. For example, $A = \{1, 3, 8\}$ is a set containing those three elements.
We may also specify a set through a condition. For example:
$A = \{x \in \mathbb{Z} \mid x \text{ is even}\}$, read as "$A$ is the set of all integers x such that x is even", is the set of all even integers.

We have the following set operations:

Suppose Ω is the universal set, and A and B are subsets of Ω.
We use the notation:
$A \subseteq \Omega$ (read as "A is a subset of Ω"), and
$B \subseteq \Omega$ (read as "B is a subset of Ω").
Likewise, if A were also a subset of B, we would write that as $A \subseteq B$.
In the same way that if $x \geq y$ and $y \geq x$ then we must have $x = y$, it follows from the definitions that if $A \subseteq B$ and $B \subseteq A$ then $A = B$.

Note that the relation of "being a subset of" is analogous to the concept "being less than or equal to." So if A is a subset of Ω, it's possible that A and Ω are equal. Analogous to "being strictly less than" is the relation "being a proper subset of."

If A is a subset of Ω and is not equal to Ω, we say A is a **proper subset** of Ω, and we write this as $A \subset \Omega$.
We denote the empty set as $\emptyset$. It is just the set which has no elements.
By $n(A)$ we denote the **cardinality** of A, which is the number of elements it contains.

Set operations

Suppose our universal set is Ω, and A and B are subsets of Ω. Then:

$A \cap B$, read as "A intersection B" is the intersection of A and B, which means it's the set of elements which are in both A and B.

$A \cup B$, read as "A union B" is the union of A and B, which means it's the set of elements which are in A, in B, or in both.

A', read as "A complement" is the complement of A, which is the set of all elements of Ω which are not in A. This is often denoted as $\Omega - A$, though we will not use this notation.

Examples

E.g. #1. Suppose $\Omega = \mathbb{Z}$, $A = \{x \in \Omega \mid 0 < x < 8\}$, $B = \{x \in \Omega \mid x \text{ is even}\}$
(a) List the elements of A.
Since Ω is the set of integers, A is the set of those integers which are strictly greater than 0 and strictly lesser than 8. So they are 1, 2, 3, 4, 5, 6, and 7.
(b) List the elements of $A \cap B$.
$A \cap B$ will be the set of those numbers which are in A and which are even. Using our answer to (a), we can see they will be 2, 4, and 6.

E.g. #2. Suppose $\Omega = \{x \in \mathbb{Z} \mid -8 < x < 8\}$, $A = \{x \in \Omega \mid x \text{ is divisible by } 3\}$, and $B = \{x \in \Omega \mid x$ is negative$\}$.
(a) Find $n(\Omega)$.
Ω is the set of all the integers from -7 to 7, which are 15. So $n(\Omega) = 15$.
(b) List the elements of B'.
B is the subset of Ω containing the negative numbers of U. So B' will contain the non-negative ones: 0, 1, 2, 3, 4, 5, 6, and 7.
(c) List the elements of $A \cap B'$.
This will contain those elements of B' which are also in A, meaning they're also divisible by 3. So it contains 0, 3, and 6.
(d) List the elements of $(A \cap B') \cup (A' \cap B)$.
We also know what $A \cap B'$ is. So we need to find $A' \cap B$. This will be those elements of Ω which are negative and aren't divisible by 3, so they're -7, -5, -4, -2, and -1.
$(A \cap B') \cup (A' \cap B)$ is the union of this set with the one we found in (c), so it will contain all the elements of both: -7, -5, -4, -2, -1, 0, 3, and 6.

E.g. #3. For each of the following, state whether or not it's true, explaining in words why:
(a) $\mathbb{I} \cap \mathbb{Q} = \emptyset$
This is true, because the set of irrationals is defined as the set of all real numbers which aren't rational. Therefore, there can't be any number which is both rational and irrational, so the intersection of the two sets is empty.
(b) $\mathbb{Z} \subset \mathbb{N}$
This is false, because there are integers which are not natural numbers. For example, -1 (or any negative integer).
(c) $\mathbb{Q} \subset \mathbb{R}$
This is true, because every rational number is also a real number (so $\mathbb{Q}$ is a subset of $\mathbb{R}$), but some real numbers aren't rational, so the two sets are not equal and $\mathbb{Q}$ is a proper subset of $\mathbb{R}$.

Homework

#H1. Suppose the following:

$\Omega = \{-9, 1, 5, 6, 8, 10\}$
$A = \{x \in \Omega | x \text{ is even}\}$
$B = \{x \in \Omega | x \geq 6\}$
$C = \{x \in \Omega | x \text{ is divisible by } 3\}$

(a) List the elements of A.
(b) List the elements of B.
(c) List the elements of C.
(d) List the elements of A'.
(e) Is A a subset of B?
(f) Is A a proper subset of B?
(g) Is C a subset of B?
(h) List the elements of $A \cap C$.
(i) List the elements of B'.
(j) List the elements of $B' \cup C$.