Practice problems and homework for Friday, Nov. 4th

Mr. Neeman. 10B, November 2, 2011.

Note: If you're having trouble factorizing the expressions in these, try doing some kind of temporary substitution (e.g. $x = \cos\alpha, y = \sin\alpha$), factorize, and then substitute them back. Remember you can always check whether a solution you find is right or not by substituting it back into the equation.

<u>Practice problems, with solutions</u>

#1. Find all the solutions of $2\sin\alpha\tan\alpha + 2\sin\alpha + \tan\alpha + 1 = 0$ in $[0, 2\pi]$
To factorize, we can substitute: $\sin\alpha = a$, $\tan\alpha = b$
$2ab + 2a + b + 1 = 0$
$(2a + 1)(b + 1) = 0$
$(2\sin\alpha + 1)(\tan\alpha + 1) = 0$
Case 1: $2\sin\alpha + 1 = 0$, so that $\sin\alpha = -\frac{1}{2}$. The corresponding preimages in $[0, 2\pi]$ are $210°$ and $330°$.
Case 2: $\tan\alpha + 1 = 0$, so that $\tan\alpha = -1$. The corresponding preimages in $[0, 2\pi]$ are $135°$ and $315°$.
So our solution set is $\{135°, 210°, 315°, 330°\}$. In this case, it's not an issue, but usually if we have tangent, secant, cosecant, or any division, we have to make sure nothing is undefined (e.g. that we don't have $\tan 90°$, or so on.

#2. Find all the solutions of $\sqrt{3}\cos\alpha = -\cos\alpha\tan\alpha$ in $[-\pi, \pi]$
Since we have $\tan\alpha$, we know that any solution must not make $\tan\alpha$ undefined. Since $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$, this just means $\cos\alpha$ cant́ be zero. In that case, we can divide both sides by $\cos\alpha$:
$\sqrt{3} = -\tan\alpha$
$-\sqrt{3} = \tan\alpha$
The corresponding images in $[-\pi, \pi]$ are $-60°$ and $120°$, so these are our solutions.
Another way would have been to factorize as usual, but then we get some solutions which make tangent undefined and which need to be discarded.

#3. Find all the solutions of $\sin^2\alpha + 5 = 4\sin\alpha$ in $[0, 2\pi]$
$\sin^2\alpha - 4\sin\alpha + 5 = 0$
Substituting $\sin\alpha = a$:
$a^2 - 4a + 5 = 0$
This has no solutions, which means the left hand side is never zero, whatever $\sin\alpha$ may be. So the original equation has no solutions.

<u>Homework</u>

#1. Find all the solutions of $4\cos^2\alpha = 3$ in the interval $[0, 2\pi]$.

#2. Find all the solutions of $1 + \cos\alpha\sin\alpha = \cos\alpha + \sin\alpha$ in $[-\pi, \pi]$.

#3. Find all the solutions of $2\cos^2\alpha + 3\cos\alpha + 1 = 0$ in $[0, 2\pi]$

#4. Express $3\sin^2\alpha + 3\cos^2\alpha - 1$ in terms of $\sin\alpha$ and constants.

#5. Express $\sec^2\alpha$ in terms of $\sin\alpha$ and constants.