Quiz #1 Solutions. Algebraic Principles 2, 10B, Fall 2011, Mr. Neeman.

#1. Consider the function $f(x) = 2x^2 - 6$.
(a) Find the equation of the axis of symmetry (3 pts).
Using the formula, the axis has equation: $x = -\frac{b}{2a}$, which in our case is $x = 0$.
Alternatively, since this is already in canonical form, you can just see the answer from the fact that it's not shifted to the left or right.

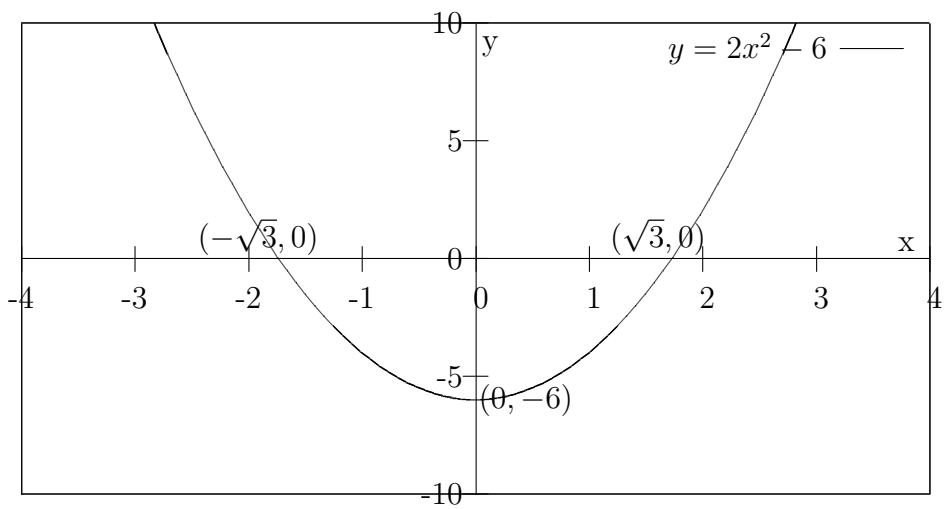
(b) Find the coordinates of the vertex (the turning point) (3 pts).
The y value for the vertex can be found by substituting in the $x = 0$ and finding y: $y = f(0) = -6$. So the vertex is at $(0, -6)$.

(c) Find the intersection with the y axis (2 pts).
The intersection with the y axis is found by substituting $x = 0$ (so it's the same as the vertex. So the only intersection with the y axis is at $(0, -6)$.

(d) Find the intersections, if there are any, with the x axis (4 pts).
To find the intersections with the x axis we solve $y = 0$, so that $2x^2 - 6 = 0$. This gives $x = -\sqrt{3}$ and $x = \sqrt{3}$. So the intersections with the x axis are $(0, -\sqrt{3})$ and $(0, \sqrt{3})$

(e) Sketch the graph of the function, including appropriately all the features you found above in (a)-(d) (8 pts).

Graph for $f(x) = 2x^2 - 6$



(f) Find the range of the function (2 pts).
The function's lowest point is at the vertex, where $f(0) = -6$, and the function then increases without bound. So the range is $[-6, \infty[$.

(g) Find the image of -3 (2 pts).
$f(-3) = 2(-3)^2 - 6 = 18 - 6 = 12$.

(h) Find the preimage or preimages of 26 (4 pts).
We set $f(x) = 26$, so $2x^2 - 6 = 26$ and $x^2 = 16$. Therefore, the preimages of 26 are -4 and 4.

(i) Find the interval in $\mathbb{R}$ in which the function is increasing (2 pts)
As seen on the graph, the function is increasing on $[0, \infty[$.

#2. Consider the function $f(x) = -x^2 - 4x - 2$.
(a) Find the equation of the axis of symmetry (3 pts).
Using the formula, the axis has equation: $x = -\frac{b}{2a}$. In our case, $-\frac{b}{2a} = -\frac{-4}{-2} = -2$, so it's $x = -2$.
This can also be done by completing the square: $f(x) = -(x+2)^2 + 2$, so the axis of symmetry is $x = -2$.

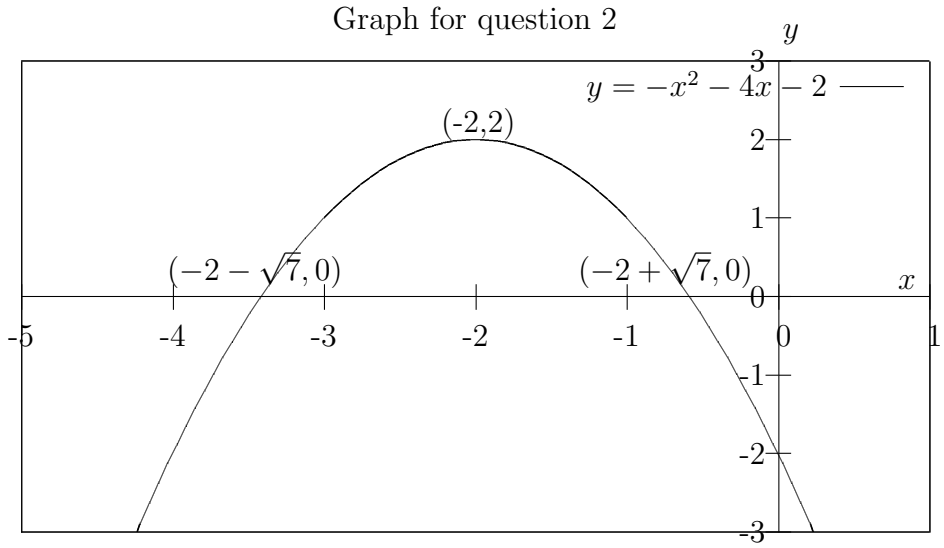
(b) Find the coordinates of the vertex (the turning point) (3 pts).
One way is to substitute $x = -2$, which gives $y = -4 + 8 - 2 = 2$. Another is, using the canonical form found in (a) to just see it's 2. So the vertex is at $(-2, 2)$.

(c) Find the intersection with the y axis (2 pts).
$f(0) = -0 - 0 - 2 = -2$. So the intersection is $(0, -2)$.

(d) Find the intersections, if there are any, with the x axis (4 pts).
We set $f(x) = 0$, which gives $-x^2 - 4x - 2 = 0$, so $x^2 + 4x + 2 = 0$. This can be solved using the general formula or by completing the square: $(x+2)^2 - 2 = 0$, so $x = -2 \pm \sqrt{2}$. So the intersections are $(-2 - \sqrt{2}, 0)$ and $(-2 + \sqrt{2}, 0)$.

(e) Sketch the graph of the function, including appropriately all the features you found above in (a)-(d) (8 pts).

Graph for question 2



(f) Find the range of the function (2 pts).
The function's highest point is at the vertex, where $f(-2) = 2$, and it goes downwards without bound. So the range is $]-\infty, 2]$.

(g) Find the image of $\frac{3}{2}$ (2 pts).
$f(\frac{3}{2}) = -(\frac{3}{2})^2 - 4\frac{3}{2} - 2 = -\frac{9}{4} - 6 - 2 = -\frac{41}{4}$.

(h) Find the preimage or preimages of -5 (4 pts).
We set $f(x) = -5$, so $-x^2 - 4x - 2 = -5$ and $x^2 + 4x - 3 = 0$. This can be solved by the general formula or by completing the square: $(x+2)^2 - 7 = 0$, so $x = -2 \pm \sqrt{7}$. Therefore the preimages of -5 are $-2 - \sqrt{7}$ and $-2 + \sqrt{7}$.

(i) Find the interval in $\mathbb{R}$ in which the function is increasing (2 pts)
The function is increasing in its left half, and the vertex is at $x = -2$, so the function is increasing on $]-\infty, -2]$.