Practice for Midterm #2. Solutions.
Mr. Neeman. 10B. September 16.

#1. Consider the function $f(x) = \log_{\frac{1}{6}} x$.

(a) Find the image of $\dfrac{1}{\sqrt{6}}$.

$\log_{\frac{1}{6}} \dfrac{1}{\sqrt{6}}$

$= \log_{\frac{1}{6}} \left(\left(\dfrac{1}{6} \right)^{\frac{1}{2}} \right)$

$= \dfrac{1}{2}$

(b) Find the image of 36.

$\log_{\frac{1}{6}} 36$

$= \log_{\frac{1}{6}} \left(\left(\dfrac{1}{6} \right)^{-2} \right)$

$= -2$

(c) Find the preimage of -1.

$\log_{\frac{1}{6}} x = -1$

$x = \left(\dfrac{1}{6} \right)^{-1}$

$x = 6$

(d) Find the preimage of $\dfrac{3}{2}$.

$\log_{\frac{1}{6}} x = \dfrac{3}{2}$

$x = \left(\dfrac{1}{6} \right)^{\frac{3}{2}}$

$x = \dfrac{1}{\sqrt{6^3}}$

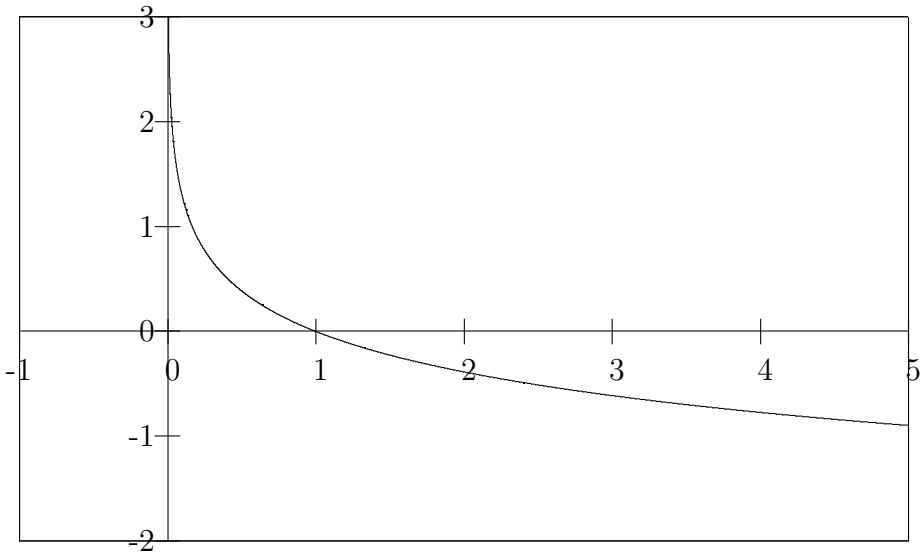
(e) Find the function's intersection with the x axis.

$\log_{\frac{1}{6}} x = 0$

$x = \left(\dfrac{1}{6} \right)^{0}$

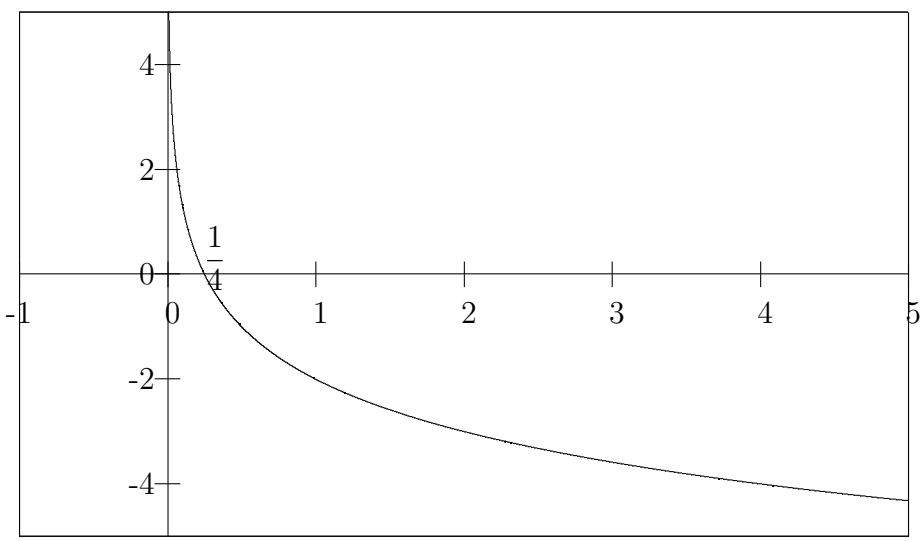
$x = 1$

(f) Sketch the function's graph, labeling your result from (c).



(g) Is the function increasing or decreasing?
Decreasing.
(h) What is the function's concavity?
Concave up.
(i) What is the equation of the function's asymptote?
$x = 0$
(j) What is the function's range?
$\mathbb{R}$
(k) What is the function's domain?
$[0, \infty[$
(l) If the function's domain was restricted to $[6, \infty[$, what would be the range?
$f(6) = \log_{\frac{1}{6}} 6 = \log_{\frac{1}{6}} \left(\left(\frac{1}{6} \right)^{-1} \right) = -1$
As x goes to ∞, $f(x)$ goes to $-\infty$. So the range will be between -1 (inclusive) and $-\infty$. We write this as:
$]-\infty, -1]$
(m) What would the domain have to be for the range to be $[-1, 1[$?
The preimage of -1 was found, in part (c), to be 6. Since -1 is included in the range, this will be included in the domain.
The preimage of 1 is $\left(\frac{1}{6} \right)^1 = \frac{1}{6}$. Since 1 isn't included in the range, this won't be included in the domain.
Therefore, the domain would be $]\frac{1}{6}, 6]$.

#2. Consider the function $f(x) = -2 - \log_2 x$.
(a) Find the image of 16.
$f(16) = -2 - \log_2 16 = -2 - \log_2(2^4) = -2 - 4 = -6$
(b) Find the image of 1.
$f(1) = -2 - \log_2 1 = -2 - 0 = -2$
(c) Find the preimage of 0.
$-2 - \log_2 x = 0$
$-2 = \log_2 x$
$x = 2^{-2}$
$x = \dfrac{1}{4}$
(d) Find the preimage of $-\dfrac{5}{2}$.
$-2 - \log_2 x = -\dfrac{5}{2}$
$\dfrac{1}{2} = \log_2 x$
$x = 2^{\frac{1}{2}}$
$x = \sqrt{2}$
(e) Find the function's intersection with the x axis.
$-2 - \log_2 x = 0$
$-2 = \log_2 x$
$x = 2^{-2}$
$x = \dfrac{1}{4}$
(f) Sketch the function's graph, labeling your result from (c).



(g) Is the function increasing or decreasing?
Decreasing.
(h) What is the function's concavity?
Concave up.

(i) What is the equation of the function's asymptote?
$x = 0$
(j) What is the function's range?
$\mathbb{R}$
(k) What is the function's domain?
$]0, \infty[$
(l) If the function's domain was restricted to $]2, 8]$, what would be the range?
$f(2) = -2 - \log_2 2 = -2 - 1 = -3$. This isn't included.
$f(8) = -2 - \log_2 2 = -2 - \log_2(2^3) = -2 - 3 = -5$. This is included.
So the range would be $[-5, -3[$.
(m) What would the domain have to be for the range to be $]-\infty, 2]$?
The function goes to $-\infty$ when x goes to ∞. And the preimage of 2 is:
$2 = -2 - \log_2 x$
$4 = -\log_2 x$
$x = 2^{-4} = \dfrac{1}{16}$
So the domain would be $[\dfrac{1}{16}, \infty[$.

#3. Express each of the following as a single logarithm.
(a) $2\log_2 x - \log_2(y^3)$
$= \log_2(x^2) - \log_2(y^3)$
$= \log_2 \dfrac{x^2}{y^3}$

(b) $\log_5 \sqrt{xy} + \log_5 \dfrac{x}{z}$
$= \log_5(\sqrt{xy}\dfrac{x}{z})$
$= \log_5 \dfrac{\sqrt{x^3y}}{z}$

(c) $\log_3(3x) - \log_9(xy^4)$
We can go to either base 3 or base 9. To go to base 3:
$= \log_3(3x) - \dfrac{\log_3(xy^4)}{\log_3 9}$
$= \log_3(3x) - \dfrac{\log_3(xy^4)}{\log_3(3^2)}$
$= \log_3(3x) - \dfrac{\log_3(xy^4)}{2}$
$= \log_3(3x) - \log_3(\sqrt{xy^4})$
$= \log_3 \dfrac{3x}{\sqrt{xy^4}}$
$= \log_3 \dfrac{3\sqrt{x}}{y^2}$

The other possibility is to switch to base 9:
$\log_3(3x) - \log_9(xy^4)$
$= \dfrac{\log_9(3x)}{\log_9 3} - \log_9(xy^4)$
$= \dfrac{\log_9(3x)}{\log_9(9^{\frac{1}{2}})} - \log_9(xy^4)$
$= \dfrac{\log_9(3x)}{\frac{1}{2}} - \log_9(xy^4)$
$= 2\log_9(3x) - \log_9(xy^4)$
$= \log_9(9x^2) - \log_9(xy^4)$
$= \log_9 \dfrac{9x^2}{xy^4}$
$= \log_9 \dfrac{9x}{y^4}$

(d) $\log_4(3x) - \log_4(y^{-3}) + 2\log_4(xy^2)$
$= \log_4 \dfrac{3x}{y^{-3}} + 2\log_4(xy^2)$
$= \log_4 \dfrac{3x}{y^{-3}} + \log_4(x^2y^4)$
$= \log_4(3xy^3) + \log_4(x^2y^4)$
$= \log_4((3xy^3)(x^2y^4))$
$= \log_4(3x^3y^7)$
#4. Express each of the following in terms of $\log_2 x$ and $\log_2 y$ (constants are ok, of course).
(a) $\log_8(xy^2)$
$= \log_8 x + \log_8(y^2)$
$= \log_8 x + 2\log_8 y$

(b) $\log_2 \sqrt{x^3y^{-1}}$
$= \log_2(x^{\frac{3}{2}}y^{-\frac{1}{2}})$
$= \log_2(x^{\frac{3}{2}}) + \log_2(y^{-\frac{1}{2}})$
$= \dfrac{3}{2}\log_2 x - \dfrac{1}{2}\log_2 y$

(c) $\log_2 \dfrac{xy}{\sqrt{x}} - \log_2 x$
$= \log_2(xy) - \log_2\sqrt{x} - \log_2 x$
$= \log_2 x + \log_2 y - \log_2\sqrt{x} - \log_2 x$
$= \log_2 y - \log_2\sqrt{x}$
$= \log_2 y - \dfrac{1}{2}\log_2 x$

(d) $\log_4 x - \log_{\frac{1}{2}}(xy)$
$= \log_4 x - \dfrac{\log_4(xy)}{\log_4 \frac{1}{2}}$
$= \log_4 x - \dfrac{\log_4(xy)}{\log_4(4^{-\frac{1}{2}})}$

$= \log_4 x - \dfrac{\log_4(xy)}{-\frac{1}{2}}$
$= \log_4 x + 2\log_4(xy)$
$= \log_4 x + 2\log_4 x + 2\log_4 y$
$= 3\log_4 x + 2\log_4 y$

#5. Solve each of the following equations.

(a) $\left(\dfrac{1}{3}\right)^{5x+2} - 4 = 8$

$\left(\dfrac{1}{3}\right)^{5x+2} = 12$

$5x + 2 = \log_{\frac{1}{3}} 12$

$x = \dfrac{\log_{\frac{1}{3}} 12 - 2}{5}$

(b) $\log_4(x^3 + 2) = 2$

$x^3 + 2 = 4^2$

$x^3 = 14$

$x = \sqrt[3]{14}$

We can check that this does make $x^3 + 2$ positive, so it is a solution.

(c) $\log_3 x + \log_3(x - 5) = \log_3 14$

$\log_3(x(x - 5)) = \log_3 14$

(note: one could apply the exponential function at this point, as in part (d). Either way works.)

$\log_3(x(x - 5)) - \log_3 14 = 0$

$\log_3 \dfrac{x(x - 5)}{14} = 0$

$\dfrac{x(x - 5)}{14} = 3^0$

$\dfrac{x(x - 5)}{14} = 1$

$x(x - 5) = 14$

$x^2 - 5x - 14 = 0$

$(x + 2)(x - 7) = 0$

So $x = -2$ or $x = 7$.

$x = -2$ isn't a solution, because it's negative, so $\log_3 x$ would be undefined. On the other hand, if $x = 7$, both x and $x - 5$ are positive, so it is a solution. Therefore, $x = 7$.

(d) $\log_5 x - \log_5(x + 2) - 1 = 0$

$\log_5 x - \log_5(x + 2) = 1$

$\log_5 \dfrac{x}{x + 2} = 1$

$\dfrac{x}{x + 2} = 5^1$

$\dfrac{x}{x + 2} = 5$

$x = 5x + 10$

$-10 = 4x$

$x = -\frac{5}{2}$

However, this makes $\log_5 x$ undefined (and $\log_5(x+2)$ would also be undefined), so it's not a solution.

Therefore, there are no solutions.