Maximal domains

Mr. Neeman. 10A, September 14, 2011

Given a mapping, we can often figure out which values it can take as inputs and which it can't. The set of all the inputs it can take is called the **maximal domain**. For now, we will be considering rational functions (which is a technical term for functions with polynomials and fractions with polynomials) and radical functions (ones with roots).

E.g. #1. $f(x) = \sqrt{x}$.

As we've seen before, this can't take in any negative numbers, since the square root of a negative number isn't defined. And it can take in 0 or any positive number. So the maximal domain is $[0, \infty]$.

E.g. #2. $f(x) = \sqrt{1-x}$

For $f(x)$ to be defined, we must have $1 - x \geq 0$, since we can't take the square root of a negative number. So we have $1 \geq x$, so that our maximal domain is $]-\infty, 1]$.

E.g. #3. $f(x) = \dfrac{x^2}{x^2 - 1}$

This will be defined so long as the denominator isn't 0. So we want find out when it is 0. To do this, we factorize it:

$f(x) = \dfrac{x^2}{(x+1)(x-1)}$

So it will be undefined when $x = -1$ and when $x = 1$. Then the maximal domain is all real numbers except those. This can be written as $\mathbb{R} - \{-1, 1\}$, or as $]-\infty, -1[\cup]-1, 1[\cup]1, \infty[$

E.g. #4. $f(x) = \dfrac{x}{x+3} - \dfrac{1}{3x^2}$

This will be defined if neither of the denominators is 0. So we need $x + 3 \neq 0$ and $3x^2 \neq 0$. This means $x \neq -3$ and $x \neq 0$. So the maximal domain is $\mathbb{R} - \{-3, 0\}$

E.g. #5. $f(x) = \sqrt{x+2} - \sqrt{5-x}$

Here we need to have $x + 1 \geq 0$ and $5 - x \geq 0$. This means $x \geq -1$ and $5 \geq x$, so $-1 \leq x \leq 5$, so that the maximal domain is $[-1, 5]$.

E.g. #6. $f(x) = \sqrt{x^2 - 4}$

Here we need to have $x^2 - 4 \geq 0$. This is a quadratic inequality, which we did earlier in the semester. To solve it, we can proceed to find the key values and do a table. When we do that, we find the solution consists of $x \leq -2$ and $x \geq 2$. So the maximal domain is $]-\infty, -2] \cup [2, \infty[$.

E.g. #7. $f(x) = \sqrt{2x+4} - \dfrac{x+3}{x}$

Here we have both a division and a square root.

What's inside the square root must be at least 0, so we get $2x + 4 \geq 0$, which means $x \geq -2$. This is our first restriction.

We can't divide by zero, so $x \neq 0$.

Putting these together, the maximal domain will consist of all numbers from -2 and above, except 0. We can write this as $[-2, \infty[-\{0\}$.

E.g. #8. $f(x) = \sqrt{x} + \sqrt{x-3} + \dfrac{1}{x-1}$
Here we have three restrictions:
First, since we have $\sqrt{x}$, we must have $x \geq 0$.
Second, since we have $\sqrt{x-3}$, we must have $x - 3 \geq 0$, so $x \geq 3$.
Third, since there's a division by $x - 1$, we must have $x - 1 \neq 0$, so that $x \neq 1$.
Putting these together, we find the maximal domain is $[3, \infty[$. This is because of the second condition is satisfied, the others automatically are (if $x \geq 3$ then x already can't be 1 and can't be negative).

Homework for Friday (to be begun in class)

#1. Find the maximal domain for each of the following mappings:
(a) $f(x) = \sqrt{2x+5}$
(b) $f(x) = \dfrac{1}{x-6} - \dfrac{x^2+3}{x^2+1}$
(c) $f(x) = 5 + \sqrt{9 - x^2}$
(d) $f(x) = \dfrac{x^2+6x}{x^2+7x+10}$
(e) $f(x) = \sqrt{3-x} - \dfrac{x^2+1}{2x}$
(f) $f(x) = \sqrt{3x+6} - \sqrt{x}$
#2. Solve the following inequalities:
(a) $x(x^2+2x+8)(x-5) > 0$
(b) $\dfrac{x+3}{x^2-4} \leq 0$
(c) $x^2 + 6x + 9 \leq 0$