Practice for midterm #2.
Mr. Neeman, 10A. September 16.

#1. Solve each of the following inequalities:
(a) $x^2(x-1) \leq 0$
The key values are $x = 0$ and $x = 1$.

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$x > 1$
x	-	0	+	+	+
x^2	+	0	+	+	+
$x - 1$	-	-	-	0	+
$x^2(x-1)$	-	0	-	0	+

We're looking for it to be 0 or negative, which means all the columns except the last one. So the answer is $x \leq 1$.

(b) $x^2 + 3x - 18 > 0$
$(x+6)(x-3) > 0$
The key values are $x = -6$ and $x = 3$

	$x < -6$	$x = -6$	$-6 < x < 3$	$x = 3$	$x > 3$
$x + 6$	-	0	+	+	+
$x - 3$	-	-	-	0	+
$(x+6)(x-3)$	+	0	-	0	+

We're looking for it to be positive, which means the first and last columns. So the answer is $x < -6$ and $x > 3$.

(c) $\dfrac{6}{x+4} \geq 1$
$1 - \dfrac{6}{x+4} \geq 0$
$\dfrac{x-2}{x+4} \leq 0$
The key values are $x = -4$ and $x = 2$

	$x < -4$	$x = -4$	$-4 < x < 2$	$x = 2$	$x > 2$
$x + 4$	-	0	+	+	+
$\dfrac{1}{x+4}$	-	undef.	+	+	+
$x - 2$	-	-	-	0	+
$\dfrac{x-2}{x+4}$	+	undef.	-	0	+

We're looking for it to be 0 or negative, which means $-4 < x \leq 2$.

(d) $2x^2 + 12x + 18 > 0$
$x^2 + 6x + 9 > 0$ (since 2 is positive, the inequality sign stays the same)
$(x+3)^2 > 0$
So the key value is $x = -3$.

	$x < -3$	$x = -3$	$x > -3$
$x+3$	-	0	+
$(x+3)^2$	+	0	+

We're looking for it to be positive, so it's the first and third columns, so $x < -3$ and $x > 3$.

(e) $\dfrac{x^2 + 5x + 10}{x - 1} \geq 0$
Completing the square in the numerator:
$\dfrac{(x + \frac{5}{2})^2 + \frac{15}{4}}{x - 1} \geq 0$
So we see the numerator is always positive and so has no roots. So we can divide both sides by it, giving us:
$\dfrac{1}{x-1} \geq 0$
The key value is $x = 1$.

	$x < 1$	$x = 1$	$x > 1$
$x-1$	-	0	+
$\dfrac{1}{x-1}$	-	undef.	+

We're looking for it to be 0 or positive, so our answer is $x > 1$.

#2. For each of the following, circle the type of proposition it is, or "NP" if it's not a proposition.

(a) Asia is a city in Texas. *Contingent*

(b) Either it will rain tomorrow or it won't. *Tautology*

(c) Honk if you like puppies. *Not a proposition*

(d) Peter the Great was Russian and Peter the Great wasn't Russian. *Contradiction*

(e) If it's cloudy, then it's raining. *Contingent*

(f) Bill Monroe is dead. *Contingent*

#3. Translate each of the following, using the key given

p: Alice likes painting.
q: Alice likes quilting.
r: Alice likes reading.

(a) If Alice doesn't like painting, then she likes reading.
$\neg p \Rightarrow r$

(b) Either Alice doesn't like painting or she doesn't like quilting.
$\neg p \vee \neg q$

(c) Alice likes reading if and only if she likes both painting and quilting.
$r \Leftrightarrow (p \wedge q)$

(d) Alice doesn't like quilting, and she does like reading.
$\neg q \wedge r$

(e) If Alice likes either reading or quilting, then she doesn't like quilting.
$(r \vee q) \Rightarrow \neg q$

(f) $p \veebar r$
Either Alice likes painting or she likes reading, but not both.

(g) $p \vee \neg q$
Either Alice likes painting or she doesn't like quilting.

(h) $\neg r \Rightarrow (p \wedge q)$
If Alice doesn't like reading then she likes both painting and quilting.

#4. For each of the following, draw a truth table and use it to say whether the proposition is a tautology, a contradiction, or contingent.
(a) $p \Leftrightarrow (\neg p \Rightarrow p)$

p	$\neg p$	$\neg p \Rightarrow p$	$p \Leftrightarrow (\neg p \Rightarrow p)$
T	F	T	T
F	T	F	T

It is a tautology.

(b) $p \vee (p \Rightarrow q)$

p	q	$p \Rightarrow q$	$p \vee (p \Rightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

It is a tautology.

(c) $p \veebar (\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \veebar (\neg p \wedge \neg q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	F	T	T	T	T

It is contingent.

(d) $(p \wedge q) \Leftrightarrow \neg p$

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \Leftrightarrow \neg p$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	F
F	F	F	T	F

It is is contingent.

(e) $(\neg p \vee \neg q) \Leftrightarrow (p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$(\neg p \vee \neg q) \Leftrightarrow (p \wedge q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

It is a contradiction.

#5. Use truth tables to find out, for each of the following pairs, whether the two sentences are logically equivalent or not.

(a) $p \Rightarrow \neg q$ and $\neg(p \wedge q)$

p	q	$\neg q$	$p \Rightarrow \neg q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	T

They are logically equivalent.

(b) $p \veebar \neg q$ and $p \Rightarrow q$

p	q	$\neg q$	$p \veebar \neg q$	$p \Rightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	F	F	T
F	F	T	T	T

They are not logically equivalent.

(c) $\neg(p \wedge \neg q)$ and $\neg p \wedge q$

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg p$	$\neg p \wedge q$
T	T	F	F	T	F	F
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	F

They are not logically equivalent.

#6. Write down the inverse, converse, and contrapositive for each of the following:

(a) If the moon is made of cheese, then I don't like pizza.
Inverse: If the moon isn't made of cheese, then I like pizza.
Converse: If I don't like pizza, then the moon is made of cheese.
Contrapositive: If I like pizza, then the moon isn't made of cheese.

(b) If Alice voted for Lincoln, then she is dead.
Inverse: If Alice didn't vote for Lincoln, then she isn't dead.
Converse: If Alice is dead, then she voted for Lincoln.
Contrapositive: If Alice isn't dead, then she didn't vote for Lincoln.

(c) $\neg p \Rightarrow (p \vee q)$
Inverse: $p \Rightarrow \neg(p \vee q)$
Converse: $(p \vee q) \Rightarrow \neg p$
Contrapositive: $\neg(p \vee q) \Rightarrow p$

(d) $p \Rightarrow \neg q$
Inverse: $\neg p \Rightarrow q$
Converse: $\neg q \Rightarrow p$
Contrapositive: $q \Rightarrow \neg p$

#8. Consider the following function:
$f : \mathbb{R} \to \mathbb{R}$
$f(x) = x^2 + 3$

(a) Find the image of 2.
$f(2) = 2^2 + 3 = 7$

(b) Find the image of -5.
$f(-5) = (-5)^2 + 3 = 28$

(c) Find any preimages of 0.
$f(x) = 0$
$x^2 + 3 = 0$
This has no solutions, so 0 has no preimages.

(d) Find any preimages of 3.
$f(x) = 3$
$x^2 + 3 = 3$
$x^2 = 0$
Therefore, $x = 0$
So 3 has one preimage: 0.

(e) Find any preimages of 12.
$f(x) = 12$
$x^2 + 3 = 12$
$x^2 = 9$
$x = \pm 3$
So 12 has two preimages: -3 and 3.

#9. Say whether each of the following is a function or not.
(a) $f : \mathbb{R} \to \mathbb{R}$
$f(x) = \dfrac{6}{x+2}$
This isn't a function, since -2 is in the domain, but if you give -2 as an input it is undefined (it's a division by 0).

(b) $f : \mathbb{R} \to \mathbb{R}$
$f(x) = x^2 - 9$
This is a function.

(c) $f : [6, \infty[\to \mathbb{R}$
$f(x) = \sqrt{x-4} + \sqrt{x}$
This is a function, since the domain only includes numbers from 6 and up, for which both $x - 4$ and x are positive.

#10. Find the maximal domain for each of the following mappings:
(a) $f(x) = 5x - 19$
This is defined for any number, so the maximal domain is $\mathbb{R}$

(b) $f(x) = 2\sqrt{x-8}$
We need what's inside the square root to be at least 0:
$x - 8 \geq 0$
$x \geq 8$
So the maximal domain is $[8, \infty[$

(c) $f(x) = \dfrac{3x+4}{x^2}$ We can't divide by zero, so we must have
$x^2 \neq 0$
Now, x^2 is zero if and only if $x = 0$, so we get:
$x \neq 0$
So the maximal domain is $\mathbb{R} - \{0\}$
(d) $f(x) = \sqrt{x-3} - 3\sqrt{10-x}$
We need what's inside each square root to be at least zero. So we get:
$x - 3 \geq 0$ and
$10 - x \geq 0$
The first gives us $x \geq 3$ and the second gives $10 \geq x$. So the maximal domain is $[3, 10]$.

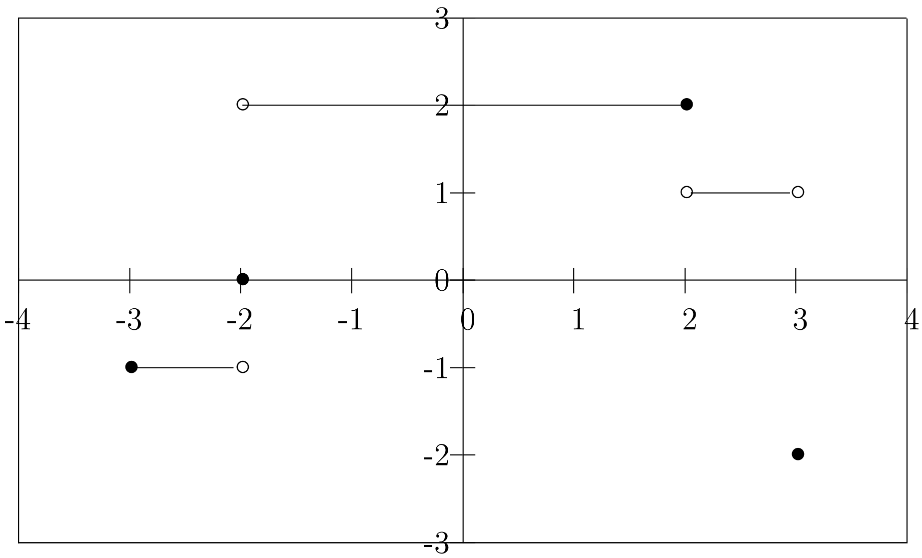
(e) $f(x) = \dfrac{\sqrt{2x-1}}{x+4}$
We need what's inside the square root to be at least 0, and the denominator to not be 0. So we get:
$2x - 1 \geq 0$ and
$x + 4 \neq 0$
The first one gives us $x \geq \dfrac{1}{2}$ and the second gives us $x \neq -4$. However, the second is automatically satisfied if the first is satisfied, since if $x \geq \dfrac{1}{2}$ then x can't be -4.
So the maximal domain is $[\dfrac{1}{2}, \infty[$.

(f) $f(x) = \sqrt{x+4} + \dfrac{1}{x-1}$
We need what's inside the square root to be at least 0, and the denominator to not be 0. So we get:
$x + 4 \geq 0$ and
$x - 1 \neq 0$
The first one gives us $x \geq -4$ and the second gives us $x \neq 1$.
So the maximal domain is $[-4, \infty[-\{1\}$.

#11. Consider the following function:
$f : [-3, 3] \to \mathbb{R}$

$$f(x) = \begin{cases} -1 & \text{if } -3 \leq \text{x} < -2 \\ 0 & \text{if x} = -2 \\ 2 & \text{if } -2 < \text{x} \leq 2 \\ 1 & \text{if } 2 < \text{x} < 3 \\ -2 & \text{if x} = 3 \end{cases}$$

(a) Sketch the function's graph.



(b) Find the image of 1.
$f(1) = 2$

(c) Find the image of -2.
$f(-2) = 0$

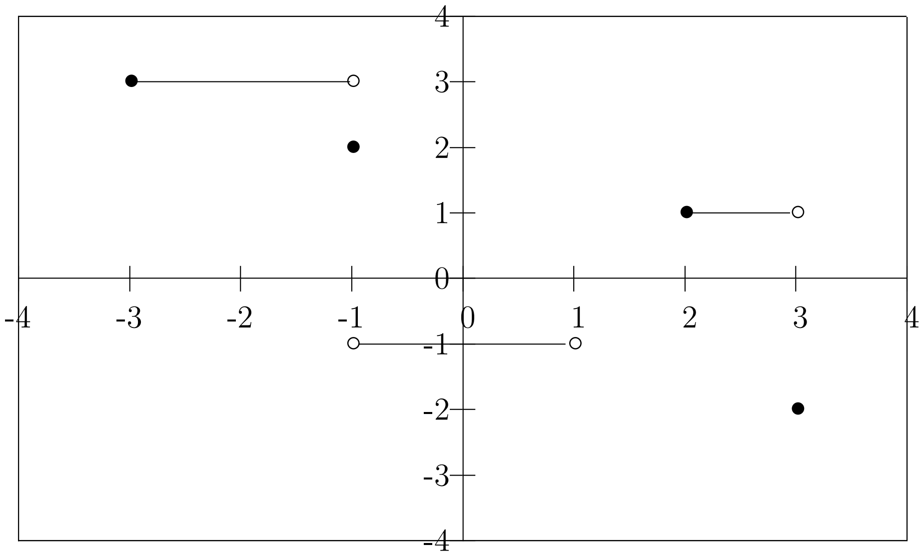
(d) Find the image of 2.
$f(2) = 2$

(e) Find any preimages of 2
$-2 < x \leq 2$.
Another way to put it is that the set of preimages is $]-2, 2]$.

(f) Find any preimages of -2.
$x = 3$

(g) Find any preimages of 3.
It has no preimages.

#12. Consider this graph of the function f, with codomain $\mathbb{R}$.



(a) Find the function's domain.
$[-3, 1[\cup[2, 3]$
(b) Write down the function's mapping.

$$f(x) = \begin{cases} 3 & \text{if } -3 \leq \text{x} < -1 \\ 2 & \text{if x} = -1 \\ -1 & \text{if } -1 < \text{x} < 1 \\ 1 & \text{if } 2 \leq \text{x} < 3 \\ -2 & \text{if x} = 3 \end{cases}$$

(c) Find the image of -1.
$f(-1) = 2$

(d) Find the image of 2.
$f(2) = 1$

(e) Find the image of -2.
$f(-2) = 3$

(f) Find any preimages of 1.
$2 \leq x < 3$, or $[2, 3[$

(g) Find any preimages of -2.
$x = 3$

(h) Find any preimages of -3.
It has no preimages.