

Practice for Oct. 25th midterm.
Mr. Neeman, 10A. Oct. 21st, 2011.

#1. Consider the function $f(x) = 1 - (x - 2)^2$.

- (a) Use a table of values to sketch the function's graph.
- (b) Find the function's intersections with the axes.
- (c) What is the function's range?
- (d) Say whether the function is injective or not, surjective or not, and bijective or not.
- (e) Where is the function's increasing and where is it decreasing?
- (f) Find the image of -1 .
- (g) Find any preimages of 3 .
- (h) Find $f(x + 3)$.
- (i) Describe, in words, the relationship between the graph of $f(x)$ and the graph of $f(x + 3)$.
Use this to sketch the graph of $f(x + 3)$
- (j) Find $f(-x)$.
- (k) What is the function's concavity?

#2. Consider the function $f(x) = \frac{1}{2}x - 4$.

- (a) Use a table of values to sketch the function's graph.
- (b) Find the function's intersections with the axes.
- (c) What is the function's range?
- (d) Say whether the function is injective or not, surjective or not bijective or not.
- (e) What is the function's monotonicity?
- (f) Find the image of $-\frac{1}{2}$.
- (g) Find any preimages of 5 .
- (h) Find $f(z^2)$
- (i) Find $f(2 + 4x)$.

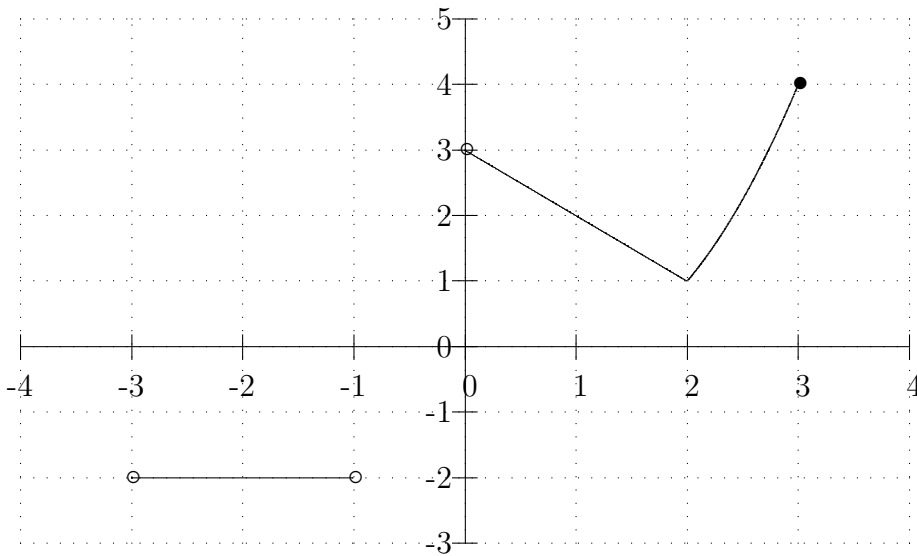
#3. Describe, in words, how the graph of each of the following looks compared to the graph of $f(x)$.

- (a) $-f(x - 1)$
- (b) $6 + f(-x)$
- (c) $-f(-x)$
- (d) $3f(x + 1)$
- (e) $f(\frac{1}{2}x)$
- (f) $f(2x) - 3$

#4. Suppose $f(x) = x - x^2$ and $g(x) = \frac{2}{x}$. Find:

- (a) $fg(x)$
- (b) $fg(1 - x)$
- (c) $fg(-1)$
- (d) $gf(x)$
- (e) $gf(2)$
- (f) $gf(z^2)$

#5. Consider the function whose graph is shown below.



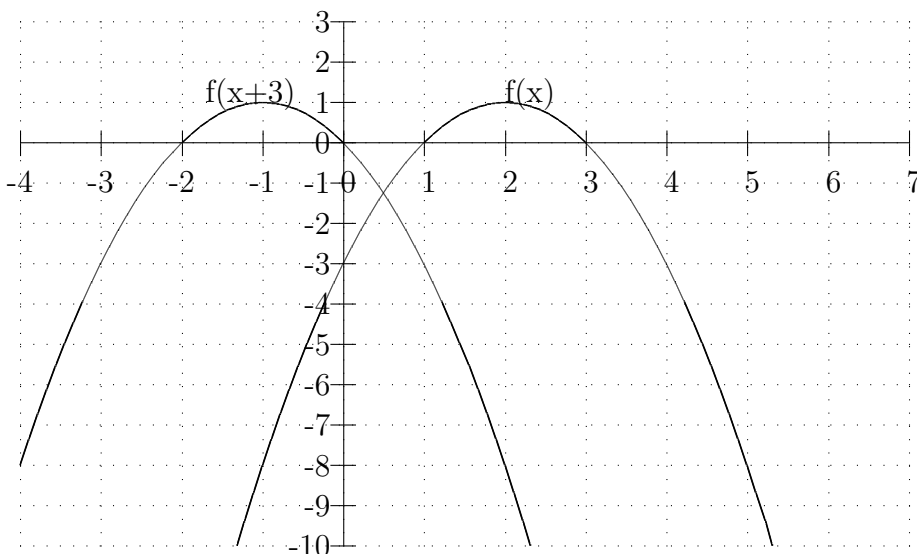
- (a) What is the function's domain?
- (b) What is the function's range?
- (c) What is the function's monotonicity on the interval $[0, 2]$?
- (d) Find the image of 1.
- (e) Find any preimages of 1.
- (f) Find any preimages of -1.
- (g) Is the function injective?
- (h) Is the function surjective?
- (i) Is the function bijective?

For additional practice on graphing transformations of functions, see the other handout for Oct. 21st

Solutions

#1. Consider the function $f(x) = 1 - (x - 2)^2$.

(a) Use a table of values to sketch the function's graph.



(b) Find the function's intersections with the axes.

For the intersection with the y axis, we set $x = 0$.

$$f(0) = 1 - (-2)^2 = 1 - 4 = -3$$

For the intersections with the x axis, we set $y = 0$

$$1 - (x - 2)^2 = 0$$

$$1 = (x - 2)^2$$

$$\pm 1 = x - 2$$

$$2 \pm 1 = x$$

Therefore, they're at $x = 1$ and $x = 3$.

(c) What is the function's range?

From the graph, we see the function goes up to $f(2) = 1$ and down to negative infinity, so the range is $]-\infty, 1]$.

(d) Say whether the function is injective or not, surjective or not, and bijective or not.

It's not injective, surjective, or bijective.

(e) Where is the function's increasing and where is it decreasing?

It's increasing on $]-\infty, 2]$ and decreasing on $[2, \infty[$.

(f) Find the image of -1 .

$$f(-1) = 1 - (-1 - 2)^2 = 1 - 9 = -8$$

(g) Find any preimages of 3.

There aren't any. We can see this on the graph, or by setting $f(x) = 3$:

$$1 - (x - 2)^2 = 3$$

$$-2 = (x - 2)^2$$

But the right hand side is a square, so it's greater or equal to zero, and can't ever be -2 .

(h) Find $f(x + 3)$.

$$f(x + 3) = 1 - ((x + 3) - 2)^2 = 1 - (x + 1)^2$$

(i) Describe, in words, the relationship between the graph of $f(x)$ and the graph of $f(x+3)$.

Use this to sketch the graph of $f(x+3)$

The graph of $f(x+3)$ is the graph of $f(x)$ translated 3 units to the left.

(j) Find $f(-x)$.

$$f(-x) = 1 - (-x - 2)^2$$

Notice that this can be simplified:

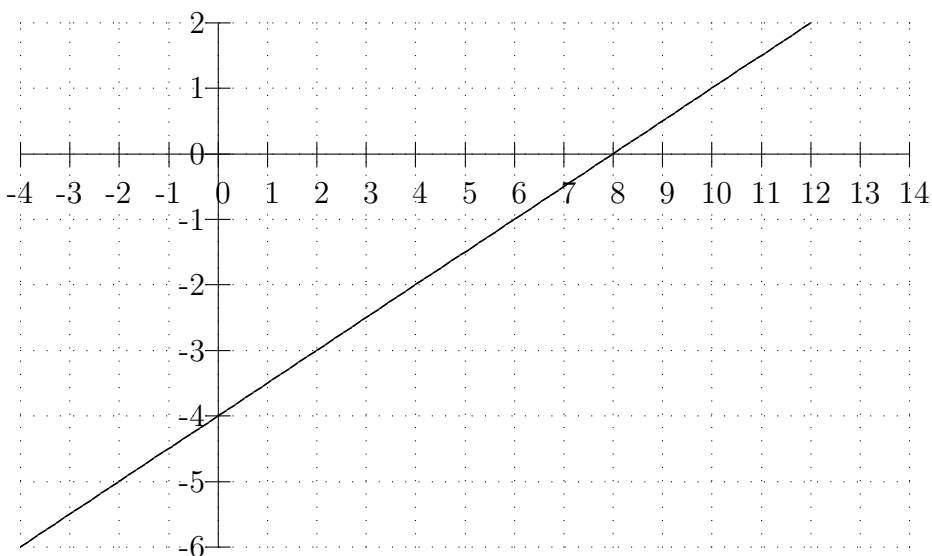
$$f(-x) = 1 - (-(x+2))^2 = 1 - (-1)^2(x+2)^2 = 1 - (x+2)^2$$

(k) What is the function's concavity?

Concave down.

#2. Consider the function $f(x) = \frac{1}{2}x - 4$.

(a) Use a table of values to sketch the function's graph.



(b) Find the function's intersections with the axes.

For the intersection with the y axis, we set $x = 0$, which gives:

$$f(0) = -4.$$

For the intersection with the x axis, we set $y = 0$, which gives:

$$\frac{1}{2}x - 4 = 0$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

(c) What is the function's range?

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(d) Say whether the function is injective or not, surjective or not bijective or not.

It is injective, surjective, and bijective.

(e) What is the function's monotonicity?

It is increasing (also, strictly increasing).

(f) Find the image of $-\frac{1}{2}$.

$$f\left(-\frac{1}{2}\right) = \frac{1}{2}\left(-\frac{1}{2}\right) - 4 = -\frac{1}{4} - 4 = -\frac{17}{4}$$

(g) Find any preimages of 5.

$$\frac{1}{2}x - 4 = 5$$

$$\frac{1}{2}x = 9$$

$$x = 18$$

(h) Find $f(z^2)$

$$f(z^2) = \frac{1}{2}z^2 - 4$$

(i) Find $f(2 + 4x)$.

$$f(2 + 4x) = \frac{1}{2}(2 + 4x) - 4 = 1 + 2x - 4 = 2x - 3$$

#3. Describe, in words, how the graph of each of the following looks compared to the graph of $f(x)$.

(a) $-f(x - 1)$ is $f(x)$ translated 1 unit to the right and then reflected vertically around the x axis.

(b) $6 + f(-x)$ is $f(x)$ reflected horizontally around the y axis and then translated 6 units up.

(c) $-f(-x)$ is $f(x)$ reflected horizontally around the y axis and then reflected vertically around the x axis.

(d) $3f(x + 1)$ is $f(x)$ translated 1 unit to the left and then stretched vertically by a factor of 3 around the x axis.

(e) $f(\frac{1}{2}x)$ is $f(x)$ stretched horizontally by a factor of 2 around the y axis.

(f) $f(2x) - 3$ is $f(x)$ compressed horizontally by a factor of 2 around the y axis and then translated down by 3 units.

#4. Suppose $f(x) = x - x^2$ and $g(x) = \frac{2}{x}$. Find:

(a) $fg(x)$

$$fg(x) = f(g(x)) = f\left(\frac{2}{x}\right) = \frac{2}{x} - \left(\frac{2}{x}\right)^2$$

(b) $fg(1 - x)$

$$fg(1 - x) = \frac{2}{1-x} - \left(\frac{2}{1-x}\right)^2$$

(c) $fg(-1)$

$$fg(-1) = \frac{2}{-1} - \left(\frac{2}{-1}\right)^2 = -2 - 4 = -6$$

(d) $gf(x)$

$$gf(x) = g(f(x)) = g(x - x^2) = \frac{2}{x-x^2}$$

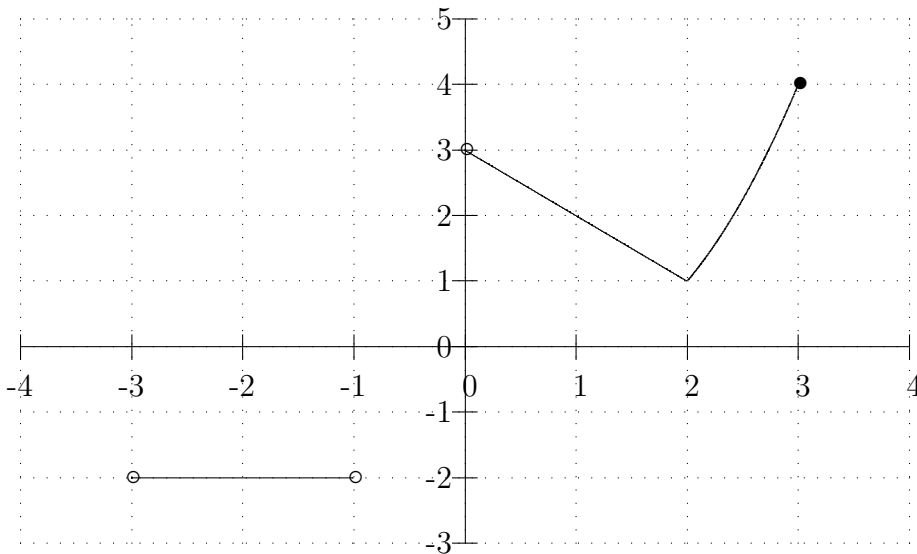
(e) $gf(2)$

$$gf(2) = \frac{2}{2-2^2} = \frac{2}{-2} = -1$$

(f) $gf(z^2)$

$$gf(z^2) = \frac{2}{z^2-z^4}$$

#5. Consider the function whose graph is shown below.



(a) What is the function's domain?

$]-3, -1[\cup]0, 3]$

(b) What is the function's range?

$\{-2\} \cup [1, 4]$

(c) What is the function's monotonicity on the interval $[0, 2]$?

It is increasing (also, strictly increasing).

(d) Find the image of 1.

Its image is 2.

(e) Find any preimages of 1.

The only preimage is 2.

(f) Find any preimages of -1.

There aren't any.

(g) Is the function injective?

No.

(h) Is the function surjective?

No.

(i) Is the function bijective?

No.