

Review: expansion and factorization

Mr. Neeman. 10A, July 25, 2011

Solutions from previous day:

Expand (multiply out) each of the following.

$$\#1. (x^2 + 3)(2x - 2) = 2x^3 - 2x^2 + 6x - 6$$

$$\#2. \left(\frac{1}{x} + y + 1\right)(x - 3) = \frac{x}{x} + xy + x - \frac{3}{x} - 3y - 3 = xy + x - 3y - 2 - \frac{3}{x}$$

$$\#3. (x + xy)(2x + \frac{5}{2}) = 2x^2 + 2x^2y + \frac{5x}{2} + \frac{5xy}{2}$$

$$\#4. (x + 2)(x + 3)(x - 1) = (x^2 + 5x + 6)(x - 1) = x^3 + 5x^2 + 6x - x^2 - 5x - 6 = x^3 + 4x^2 + x - 6$$

Factorization: common factor

In the simplest cases, one can find a factor which is common to all our terms. For example, in #3 above, $2x^2 + 2x^2y + \frac{5x}{2} + \frac{5xy}{2}$ one can see that all terms contain an x , and bringing it out leaves us with $x(2x + 2xy + \frac{5}{2} + \frac{5y}{2})$. In some cases, such as this one one can go on further. But, since the terms we have inside no longer have any factor in common we'll have to use a different method, such as the method of grouping (to be reviewed later). In other words, this method is in some cases a good start and makes the rest of the question simpler.

Exercises: factorize the following expressions as far as you can.

$$\#E1. 3xy^2 + 6y - 12x^2y$$

$$\#E2. y(x + 2) - 2xy(x + 2)$$

$$\#E3. z(x - 2) + 2y^2(2 - x)$$

Factorization: special product formulas

These are commonly occurring products:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2 \text{ (also known as the difference of squares product)}$$

Knowing these products, one can recognize expressions which fit the right hand side of one of these in order to factorize. For example: $x^2 + 6x + 9$ fits the first formula, so we can say $x^2 + 6x + 9 = (x + 3)^2$. What's important is to be able to identify these when encountering them, even if the terms are listed in a different order.

Exercises: factorize each of these as far as you can.

$$\#E4. x^2 + 4y^2 + 4xy$$

$$\#E5. 9x^2 - 12x + 4$$

$$\#E6. 3x^2 - 3x^2y^2$$

$$\#E7. x^2y^2 - 2xyz + z^2$$

$$\#E8. 4(x + y)^2 - x^2$$

$$\#E9. x^2(2y)^2 + 4xy + 1$$

$$\#E10. x^4y^2 - y^2$$

Homework practice exercises (not to be turned in, solutions at the end)

Expand each of the following.

#P1. $(2z - x^2)(y + x)$

#P2. $(x + 3y)^2$

#P3. $(x - y - 2)(x + 1)$

#P4. $(1 - x)(1 + 2x)$

#P5. $(5y + 10)(2x - y)$

Factorize each of the following as far as you can.

#P6. $3x^2y - 12xy + 6y^2$

#P7. $4y^2 - 20y + 25$

#P8. $x^2y^2 + 2xy + 1$

#P9. $4z^2 - 4$

#P10. $25y^2 - 10y + 1$

#P11. $4x^2z - 8z + 4$

#P12. $16x^2 - 9y^2$

Homework (to be turned in Thursday)

Expand each of the following:

#H1. $(x - 7)^2$

#H2. $(x - y + 1)(x^2 + 2)$

#H3. $(x + 2y)(y^2 - 1)$

#H4. $3(x^2 - y)^2$

#H5. $(x - 2y)(x + 2y)$

Factorize each of the following as far as you can:

#H6. $4x^2 - 1$

#H7. $x^2z^2 - 6xz + 9$

#H8. $3x^2y - 6x^2 + 3xy^2$

#H9. $2x^2 - 8xy + 8y^2$

#H10. $2y(3x - 1) + 3x(1 - 3x)$

Solutions for in-class exercises

- #E1. $3xy^2 + 6y - 12x^2y = 3y(xy + 2 - 4x^2)$
#E2. $y(x + 2) - 2xy(x + 2) = (y - 2xy)(x + 2) = y(1 - 2x)(x + 2)$
#E3. $z(x - 2) + 2y^2(2 - x) = (z - 2y^2)(x - 2)$
#E4. $x^2 + 4y^2 + 4xy = (x + 2y)^2$
#E5. $9x^2 - 12x + 4 = (3x - 2)^2$
#E6. $3x^2 - 3x^2y^2 = 3x^2(1 - y^2) = 3x^2(1 + y)(1 - y)$
#E7. $x^2y^2 - 2xyz + z^2 = (xy - z)^2$
#E8. $4(x + y)^2 - x^2 = (2(x + y) + x)(2(x + y) - x) = (3x + 2y)(x + 2y)$
#E9. $x^2(2y)^2 + 4xy + 1 = (2xy + 1)^2$
#E10. $x^4y^2 - y^2 = y^2(x^4 - 1) = y^2(x^2 + 1)(x^2 - 1) = y^2(x^2 + 1)(x + 1)(x - 1)$

Solutions for practice exercises

Expand each of the following.

- #P1. $(2z - x^2)(y + x) = 2yz + 2xz - x^2y - x^3$
#P2. $(x + 3y)^2 = x^2 + 6xy + 9y^2$
#P3. $(x - y - 2)(x + 1) = x^2 - x - xy - y - 2$
#P4. $(1 - x)(1 + 2x) = -2x^2 + x + 1$
#P5. $(5y + 10)(2x - y) = 10xy - 5y^2 + 20x - 10y$

Factorize each of the following as far as you can.

- #P6. $3x^2y - 12xy + 6y^2 = 3y(x^2 - 4x + 2y)$
#P7. $4y^2 - 20y + 25 = (2y - 5)^2$
#P8. $x^2y^2 + 2xy + 1 = (xy + 1)^2$
#P9. $4z^2 - 4 = 4(z - 1)(z + 1)$
#P10. $25y^2 - 10y + 1 = (5y - 1)^2$
#P11. $4x^2z - 8z + 4 = 4(x^2z - 2z + 1)$
#P12. $16x^2 - 9y^2 = (4x - 3y)(4x + 3y)$