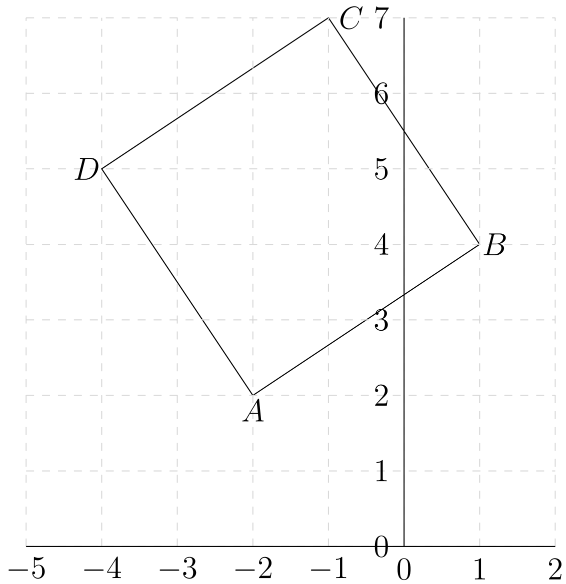
Solutions to exercises from the Nov. 11 handout.
Mr. Neeman, 10A. November 14, 2011

#1. Consider the rectangle with vertices $A = (-2, 2), B = (1, 4), C = (-1, 7)$, and $D = (-4, 5)$.
(a) Draw a labelled diagram representing the rectangle.



(b) Find the equations of each of the sides.
AB: The gradient is $\frac{2}{3}$, and it passes through $(1, 4)$. Therefore, the equation is $y = \frac{2}{3}x + \frac{10}{3}$

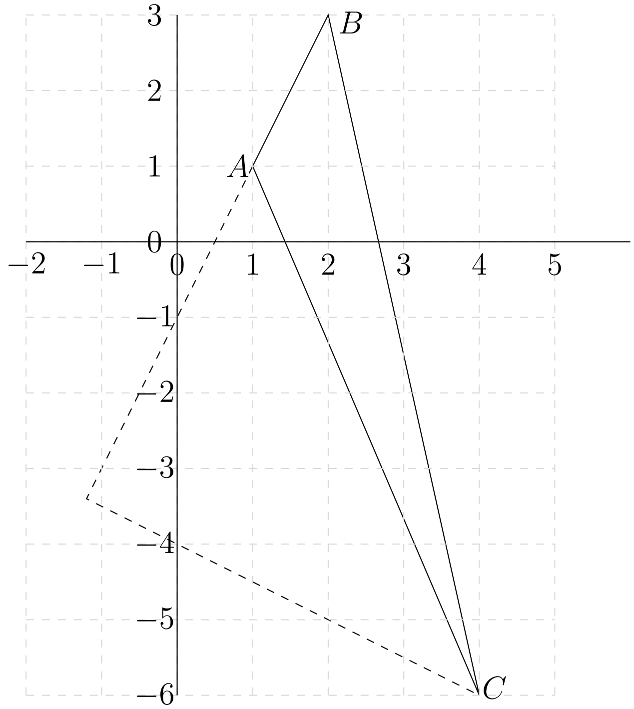
BC: The gradient is $-\frac{3}{2}$, and it passes through $(1, 4)$, Therefore, the equation is $y = -\frac{3}{2} + \frac{5}{2}$.

CD: The gradient is $\frac{2}{3}$, and it passes through $(-1, 7)$. Therefore, the equation is $y = \frac{2}{3}x + \frac{23}{3}$

DA: The gradient is $-\frac{3}{2}$, and it passes through $(-2, 2)$, Therefore, the equation is $y = -\frac{3}{2} - 1$.

(c) Verify that it is a rectangle by checking that opposite sides are parallel to each other and the pairs of adjacent sides are perpendicular to each other.
We see that AB and CD have the same gradient, that BC and DA have the same gradient, and that the gradients of AB and of BC multiplied together give -1. This means opposite sides are parallel and adjacent sides are perpendicular, so it is a rectangle.
(d) Find its area.
$AB = \sqrt{3^2 + 2^2} = \sqrt{15}$
$BC = \sqrt{2^2 + 3^2} = \sqrt{15}$ (and we see it's a square).
The area is the product of these two, which is 15.
(e) Find the length of its diagonal.
The length of the diagonal is $\sqrt{15 + 15} = \sqrt{30}$.
(f) Find the rectangle's center.
The center is the midpoint between opposite vertices. For example, between A and C, which gives us: $(-\frac{3}{2}, \frac{9}{2})$.

#2. Suppose you have a triangle with vertices $A = (1, 1), B = (2, 3)$, and $C = (4, -6)$, and that we take the side AB to be our base.
(a) Draw a labelled diagram representing this triangle.
The following diagram also includes the height, found later in the exercise.

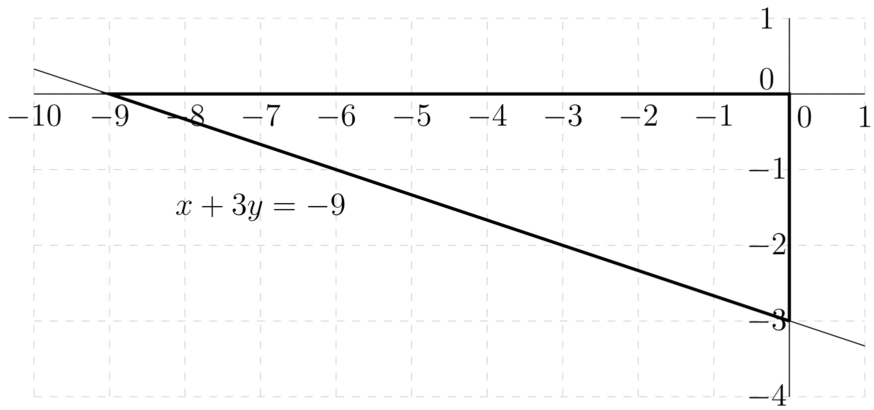


(b) Find the equation of the line containing the base.
The gradient is $\frac{2}{1} = 2$, and it passes through $(1, 1)$. So its equation will be $y = 2x - 1$.
(c) Find the equation of the line containing the height.
The height will be perpendicular to AB. Therefore, its gradient will be $-\frac{1}{2}$. It passes through $(4, -6)$. Therefore, it's equation will be $y = -\frac{1}{2}x - 4$.
(d) Find the point at which the base intersects the height.
To find the intersection of these two lines, we solve the two equations as a set. We can simply subtitute y from the first into the second:
$2x - 1 = -\frac{1}{2}x - 4$
$\frac{5}{2}x = -3$
$x = -\frac{6}{5}$.
Substituting into the first, we get $y = -\frac{12}{5} - 1 = -\frac{17}{5}$.
This is shown in the diagram
(e) Find the height.
The height is $\sqrt{(4 - (-\frac{6}{5}))^2 + (-6 - (-\frac{17}{5}))^2} = \sqrt{(\frac{26}{5})^2 + (\frac{13}{5})^2} = \frac{13}{\sqrt{5}}$
(f) Find the triangle's area.
The base is $\sqrt{1^2 + 2^2} = \sqrt{5}$. Therefore, the area is $\frac{1}{2}(\frac{13}{\sqrt{5}})(\sqrt{5}) = \frac{13}{2}$.

#3. Find the equation of a line which is perpendicular to $x = 3$ and which passes through the point $(-2, 5)$.
It will be a horizontal line, through the point $(-2, 5)$, which means it's $y = 5$.

#4. Suppose the line l_1 passes through the points $(0, 4)$ and $(-2, -1)$. Find the equation of the line which is parallel to l_1 and which passes through the point $(3, 1)$.
The gradient of l_1 is $\frac{5}{2}$. Therefore this will also be the gradient of any line parallel to l_1. Since the one we want passes through $(3, 1)$, it will have equation $y = \frac{5}{2}x - \frac{13}{2}$.

#5. Find the area of the triangle formed by the line $x + 3y = -9$ and the two axes.
A diagram here can help use see what's going on.
This line intersects the x axis at $(-9, 0)$ and the y axis at $(0, -3)$. So it looks like:



We can see it's a right-angled triangle. We can choose the long side as the base (of length 9) and the short side as the height (of length 3).
Therefore, the formula is $\frac{1}{2}(3)(9) = \frac{27}{2}$.