Polynomial equations (exercises from class and more)

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Here are the equations we've seen so far in class, plus some others. Afterwards are the answers and in some cases comments about how they can be solved.

#1. $x^3 + x^2 - 2x - 2 = 0$

#2. $x^4 - x^3 + 4x^2 - 4x = 0$

#3. $x^3 - x^2 - 10x - 8 = 0$

#4. $x^3 + 3x^2 + x - 2 = 0$

#5. $x^3 + 3x^2 + x - 2 = 0$

#6. $x^3 - 6x + 3x + 10 = 0$

#7. $2x^3 + 10x^2 + 2x - 30 = 0$

#8. $x^4 - x^2 - 12 = 0$

#9. $x^4 + 2x^3 - 2x^2 - 4x - 16 = 0$

#10. $x^3 + 4x^2 + 10x + 12 = 0$

#11. $x^4 + 4x^3 + 7x^2 - 4x - 8 = 0$

#12. $x^3 - 4x^2 - 29x + 12 = 0$

#13. $3x^4 + 3x^3 + 6x^2 + 6x = 0$

#14. $x^3 + 4x^2 - 11x - 30 = 0$

<u>Answers</u>

#1. $x^3 + x^2 - 2x - 2 = 0$

You can find the root $x = -1$ by trying the divisors of 2, and after factoring it out are left with $x^2 - 2$ which you can factorize.

$(x + 1)(x - \sqrt{2})(x + \sqrt{2}) = 0$. So $x \in \{-1, \sqrt{2}, -\sqrt{2}\}$

#2. $x^4 - x^3 + 4x^2 - 4x = 0$

You can first factor out an x, which means that $x = 0$ is a root. You can then test the divisors of 4 and see $x = 1$ works. So you can factor out $x - 1$ which leaves $x^2 + 4$. Since this quadratic has no solutions, the two solutions above are the only ones.

$x(x^2 + 4)(x - 1) = 0$. So $x \in \{0, 1\}$

#3. $x^3 - x^2 - 10x - 8 = 0$

$(x + 2)(x + 1)(x - 4) = 0$. So $x \in \{-2, -1, 4\}$

#4. $x^3 + 3x^2 + x - 2 = 0$

You can find $x = -4$, and after you factor out $x + 4$ you get $x^2 + 1$ which has no solutions.

$(x + 4)(x^2 + 1) = 0$. So $x = -4$

#5. $x^3 + 3x^2 + x - 2 = 0$

You can find $x = -2$ and after factoring out $x + 2$ you get $x^2 + x - 1$ left, which has irrational roots $\frac{1 \pm \sqrt{5}}{2}$. So $x \in \{-2, \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

#6. $x^3 - 6x + 3x + 10 = 0$
$(x-5)(x+1)(x-2) = 0$. So $x \in \{-1, 2, 5\}$

#7. $2x^3 + 10x^2 + 2x - 30 = 0$
First, you can divide both sides of the equation by 2. Then you can try the divisors of 15, finding that $x = -3$ is a solution. You can then factor out $x+3$ and have left $x^2 + 2x - 5$ which has two irrational solutions.
$2(x+3)(x+1-\sqrt{6})(x+1+\sqrt{6}) = 0$. So $x \in \{-3, -1-\sqrt{6}, -1+\sqrt{6}\}$

#8. $x^4 - x^2 - 12 = 0$
This one can be done by the usual method, trying out the divisors of 12. You find $x = 2$ and $x = -2$ are roots, and after factoring out by $x-2$ and $x+2$ you're left with a quadratic with no solutions. However, one could also take the equation as a quadratic in x^2: $(x^2)^2 - x^2 - 12 = 0$, and factorize this by inspection, getting $(x^2-4)(x^2+3) = 0$, from which you can then find the solutions.
$(x+2)(x-2)(x^2+3) = 0$. So $x \in \{-2, 2\}$.

#9. $x^4 + 2x^3 - 2x^2 - 4x - 16 = 0$
You can test the divisors of 16 to find $x = 2$ and $x = -2$ are solutions. After factoring out $x-2$ and $x+2$ you're left with $x^2 + 2x + 4$ which has no solutions.
$(x^2+2x+4)(x+2)(x-2) = 0$. So $x \in \{-2, 2\}$.

#10. $x^3 + 4x^2 + 10x + 12 = 0$
You can test the divisors to find $x = -2$ and are left with a quadratic with no solutions.
$(x+2)(x^2+2x+6) = 0$. So $x = -2$.

#11. $x^4 + 4x^3 + 7x^2 - 4x - 8 = 0$
You can test the divisors of 8 and find $x = -1$ and $x = 1$ are solutions. If you factor out the corresponding linear terms you get a quadratic without solutions.
$(x+1)(x-1)(x^2+4x+8) = 0$. So $x \in \{-1, 1\}$.

#12. $x^3 - 4x^2 - 29x + 12 = 0$
You can test the divisors to find $x = -4$ is a solution and after factoring $x+4$ out you get a quadratic with irrational roots.
$(x+4)(x-4-\sqrt{13})(x-4+\sqrt{13}) = 0$. So $x \in \{-4, 4-\sqrt{13}, 4+\sqrt{13}\}$.

#13. $3x^4 + 3x^3 + 6x^2 + 6x = 0$
Here you can begin by factoring out $3x$. This tells you either $x = 0$ or $x^3 + x^2 + 2x + 2 = 0$. For this cubic equation, you can try the divisors of 2 and find $x = -1$ is a solution. If you then divide by $x+1$ you get $x^2 + 2$ which has no solutions. You could also factorize the cubic by grouping: $x^3 + x^2 + 2x + 2 = x^2(x+1) + 2(x+1) = (x^2+2)(x+1)$.
$3x(x+1)(x^2+2) = 0$. So $x \in \{-1, 0\}$.

#14. $x^3 + 4x^2 - 11x - 30 = 0$
You can try the divisors of 30. Since there are a lot of them it's a good idea to start with the small ones and when you find one factor out the corresponding linear term.
$(x+2)(x-3)(x+5) = 0$. So $x \in \{-5, -2, 3\}$.