

Logic (part 1)

Mr. Neeman. 10A, August 30, 2011

Topics in the logic section

- #1. Basic logical concepts: proposition, tautology, contradiction, logical equivalence, inverse, converse, contrapositive.
- #2. Logical operators (not, and, or, exclusive or, if... then, if and only if).
- #3. Translations between English and logical notation.
- #4. Representing propositions using Venn diagrams.
- #5. Constructing and using truth tables

Some basic logical concepts

A **proposition** is a sentence which can be true or false. Some types of sentences which aren't propositions: questions, exclamations, orders.

Examples:

"It is raining" is a proposition.

"It will either rain tomorrow or it won't" is a proposition.

"Is it raining?" is not a proposition.

"Make it rain" is not a proposition.

"If it rains then there are clouds in the sky" is a proposition.

Propositions are classified into three types:

1. **Tautologies**: are propositions which are true as a matter of logic, regardless of any facts about the world. For example: "It will either rain tomorrow or it won't" is true regardless of what the weather is like tomorrow.
2. **Contradictions**: are propositions which are false as a matter of logic, regardless of any facts about the world. For example: "It is both raining and not raining" is false, regardless of what the weather is like.
3. **Contingent sentences**: are ones which are neither tautologies nor contradictions. They can be either true or false, depending on what the actual world is like. For example: "Costa Rica is in America" is true, but it is true in virtue of facts about the world, not due just to logic (to know Costa Rica is in America you have to know some geography, just knowing logic is not enough to tell you that).

Exercises

For each of the following sentences, figure out whether it's a proposition or not, and if it's a proposition whether it's a tautology, a contradiction, or contingent.

- #1. Don't stare at the sun.
- #2. Bill Clinton is from New York.
- #3. Bob Dole is a politician, and Bob Dole isn't a politician.
- #4. If it's raining, then it's raining.
- #5. Do you know the way to the zoo?
- #6. Either you know what's going on, or you don't.
- #7. Everyone likes Chinese food.
- #8. Alice didn't vote for Lincoln.
- #9. Did the sky just fall?
- #10. I like cheese and I don't like cheese.

Solutions

- #1. Don't stare at the sun. *not a proposition*
- #2. Bill Clinton is from New York. *contingent*
- #3. Bob Dole is a politician, and Bob Dole isn't a politician. *contradiction*
- #4. If it's raining, then it's raining. *tautology*
- #5. Do you know the way to the zoo? *not a proposition*
- #6. Either you know what's going on, or you don't. *tautology*
- #7. Everyone likes Chinese food. *contingent*
- #8. Alice didn't vote for Lincoln. *contingent*
- #9. Did the sky just fall? *not a proposition*
- #10. I like cheese and I don't like cheese. *contradiction*

Compound propositions, and translations between English and logical notation

In English, we often use sentences which can be broken down into pieces, each of which is itself grammatically a sentence. For example, from the previous exercises, #10 (and some others) are like this: "I like cheese and I don't like cheese" consists of two sentences joined by an "and". When each of them is a proposition, the entire thing is called a compound proposition.

A **compound proposition** is a proposition which contains one or more propositions as a part or parts of itself. A **simple proposition** is one which can't be decomposed into any parts which are themselves propositions. For example, "I like cheese" is a simple proposition.

When you have a compound proposition, it will be made up of simple propositions and logical operators. You have already encountered the logical operators in ordinary English, but we will have special notation for them (as well as some more technical names). We will typically represent simple sentences using letters such as p , q , and r , and give a key telling us which letters correspond to which sentences.

For the following table, suppose we use the following key:

p : I like pizza.

q : I like quiche.

Symbol	Operator	Example	Translation of example
\neg	Not/negation	I don't like pizza	$\neg p$
\wedge	And/conjunction	I like pizza and I like quiche	$p \wedge q$
\vee	Or/disjunction	Either I like pizza or I like quiche	$p \vee q$
$\underline{\vee}$	exclusive or/ exclusive disjunction	Either I like pizza or I like quiche, but not both	$p \underline{\vee} q$
\Rightarrow	If... then/implication	If I like pizza, then I like quiche	$p \Rightarrow q$
\Leftrightarrow	If and only if/ equivalence	I like pizza if and only if I like quiche	$p \Leftrightarrow q$

Examples (using the same key as above)

Sentence	Translation
Either I like quiche or I don't.	$q \vee \neg q$
If like quiche, then I don't like pizza.	$q \Rightarrow \neg p$
I either like pizza or I don't, but not both.	$p \underline{\vee} \neg p$
I like pizza if and only if I don't like quiche.	$p \Leftrightarrow \neg q$

Note: Just like there is an order of operations in arithmetic, here the \neg has priority unless there are parentheses to say otherwise. So $\neg p \wedge q$ means "I don't like pizza, and I like quiche", rather than "It's not the case that I like both pizza and quiche".

Here is the table again, for your reference.

Symbol	Operator	Example	Translation of example
\neg	Not/negation	I don't like pizza	$\neg p$
\wedge	And/conjunction	I like pizza and I like quiche	$p \wedge q$
\vee	Or/disjunction	Either I like pizza or I like quiche	$p \vee q$
$\underline{\vee}$	exclusive or/ exclusive disjunction	Either I like pizza or I like quiche, but not both	$p \underline{\vee} q$
\Rightarrow	If... then/implication	If I like pizza, then I like quiche	$p \Rightarrow q$
\Leftrightarrow	If and only if/ equivalence	I like pizza if and only if I like quiche	$p \Leftrightarrow q$

A good way to start translations from English into logical notation is by identifying any logical operators and then identifying the simple sentences.

Exercises

Use the key below to translate the given propositions between English and logical notation.

p : I like painting.

q : I like quilting.

r : I like running.

s : I like spinning.

#11. Either I like quilting or I don't like spinning.

#12. I either like quilting or like painting, but not both.

#13. If I like running, then I don't like painting.

#14. $\neg s \wedge \neg q$

#15. $\neg(p \vee r)$

#16. $q \Leftrightarrow s$

#17. $\neg s \Rightarrow \neg q$

#18. I like painting and I don't like running.

#19. If I don't like spinning, then I do like painting.

#20. Either I like running or I don't.

#21. I like spinning if and only if I don't like painting.

#22. $r \underline{\vee} s$

#23. $p \Leftrightarrow \neg r$

#24. $s \wedge \neg q$

#25. I like running if and only if I don't like running.

#26. If I don't like running, then I do like spinning.

#27. Either I like spinning or I don't like spinning, but not both.

#28. I like spinning and I like quilting.

#29. $\neg s \vee p$

#30. $\neg(p \wedge r)$

Solutions

#11. Either I like quilting or I don't like spinning.

$$q \vee \neg s$$

#12. I either like quilting or like painting, but not both.

$$q \underline{\vee} p$$

#13. If I like running, then I don't like painting.

$$r \Rightarrow \neg p$$

#14. $\neg s \wedge \neg q$

Either I don't like spinning or I don't like quilting.

#15. $\neg(p \vee r)$

It's not the case that I either like painting or like running.

#16. $q \Leftrightarrow s$

I like quilting if and only if I like spinning.

#17. $\neg s \Rightarrow \neg q$

If I don't like spinning, then I don't like quilting.

#18. I like painting and I don't like running.

$$p \wedge \neg r$$

#19. If I don't like spinning, then I do like painting.

$$\neg s \Rightarrow p$$

#20. Either I like running or I don't.

$$r \vee \neg r$$

#21. I like spinning if and only if I don't like painting.

$$s \Leftrightarrow \neg p$$

#22. $r \underline{\vee} s$

I like running if and only if I like spinning.

#23. $p \Leftrightarrow \neg r$

I like painting if and only if I don't like running.

#24. $s \wedge \neg q$

Either I like spinning or I don't like quilting.

#25. I like running if and only if I don't like running.

$$r \Leftrightarrow \neg r$$

#26. If I don't like running, then I do like spinning.

$$\neg r \Rightarrow s$$

#27. Either I like spinning or I don't like spinning, but not both.

$$s \underline{\vee} \neg s$$

#28. I like spinning and I like quilting.

$$s \wedge q$$

#29. $\neg s \vee p$

Either I don't like spinning or I like painting.

#30. $\neg(p \wedge r)$

It's not the case that I both like painting and like running.

Homework

#H1. For each of the following, state whether it's a proposition or not, and, if it's a proposition also state whether it's a tautology, a contradiction, or contingent, and whether it's compound or simple.

- (a) Everest is the highest mountain.
- (b) Either you will learn archery or you won't.
- (c) Why don't you tell me what time it is?
- (d) Owls are a type of mammal.
- (e) I know what you mean, but I don't know what you mean.
- (f) What goes up must come down.
- (g) Do what you think is right.
- (h) Not all that glitters is gold.
- (i) Either I'll like the movie or I won't.
- (j) I like pine trees and I like cedars.

#H2. Use the key below to translate the following propositions between English and logical notation.

p: Peggy voted for Lincoln.

q: The queen wore a funny hat.

r: The reverend is a good preacher.

s: Sarah can sing well.

- (a) Either Peggy voted for Lincoln or Sarah can sing well.
- (b) If the reverend is a good preacher, then the queen wore a funny hat.
- (c) If Sarah can sing well, then Peggy didn't vote for Lincoln.
- (d) Either the reverend is a good preacher, or Sarah can sing well, but not both.
- (e) The queen wore a funny hat if and only if Sarah can't sing well.
- (f) The reverend is a good preacher and the queen wore a funny hat.
- (g) $p \wedge \neg q$
- (h) $\neg q \Rightarrow \neg r$
- (i) $s \vee q$
- (j) $\neg(p \vee r)$
- (k) $r \Leftrightarrow \neg r$
- (l) If Peggy voted for Lincoln, the the queen didn't wear a funny hat.
- (m) Either the reverend is a good preacher or Peggy didn't vote for Lincoln.
- (n) The reverend is a good preacher if and only if Peggy voted for Lincoln.
- (o) Either Sarah is a good singer, or the queen wore a funny hat, but not both.
- (p) $q \vee \neg r$