

## Quadratic inequalities: homework solutions

Mr. Neeman. 10A, August 24, 2011

#H1.  $x^2 - 8x + 16 \geq 0$

$$(x - 4)^2 \geq 0$$

Now, when  $x = 4$ , the quadratic is zero, so the inequality is satisfied. And, when  $x \neq 4$ , the quadratic is positive, so the inequality is also satisfied. Therefore, the solution is all real numbers.

#H2.  $x^2 + 2x - 15 < 0$

$$(x + 5)(x - 3) < 0$$

This will be satisfied when one of these is positive and the other negative. Now, it's not possible for  $x + 5$  to be negative and  $x - 3$  positive. So it'll be satisfied when  $x + 5$  is positive, and  $x - 3$  is negative, which means  $x > -5$  and  $x < 3$ . So our answer is  $-5 < x < 3$ .

#H3.  $2x^2 + 5x - 3 \leq 0$

$$(2x - 1)(x + 3) \leq 0$$

This will be satisfied at the roots,  $x = -3$  and  $x = \frac{1}{2}$  (where the quadratic will be zero), and when one of these is positive and the other negative. It's not possible for  $x + 3$  to be negative and  $2x - 1$  to be positive, so it's the other way around:  $x + 3$  positive and  $2x - 1$  negative. This means  $x > -3$  and  $x < \frac{1}{2}$ , so that  $-3 < x < \frac{1}{2}$ . But we have to remember to add the roots, since the inequality was with less than or equal to. So  $-3 \leq x \leq \frac{1}{2}$ .

#H4.  $-x^2 + 2x - 1 > 0$

We move everything to the other side:  $0 > x^2 - 2x + 1$

$$0 > (x - 1)^2$$

This is never satisfied, since the right hand side is zero for  $x = 1$  and positive if  $x \neq 1$ . So there are no solutions.

#H5.  $x^2 + 4x - 2 \geq 0$

$$(x + 2)^2 - 6 \geq 0$$

$$(x + 2 + \sqrt{6})(x + 2 - \sqrt{6}) \geq 0$$

Our roots are, therefore,  $-2 - \sqrt{6}$  and  $-2 + \sqrt{6}$ . The quadratic will be zero at the roots, and positive when either both terms are negative or when both terms are positive. Working these out, we get  $x \leq -2 - \sqrt{6}$  or  $x \geq -2 + \sqrt{6}$ .

#H6.  $2x^2 + 8x + 12 \leq 0$

We can divide both sides by 2, and since 2 is positive the inequality sign stays the same. We get

$$x^2 + 4x + 6 \leq 0$$

$$(x + 2)^2 + 2 \leq 0$$

This has no solutions, since the left hand side is always at least 2.