Practice for the Nov. 22nd quiz
Mr. Neeman, 10A. November 18, 2011

#1. Consider the triangle formed by the line $x - 2y = 3$, the line $y = 2$, and the y axis.
(a) Find the triangle's 3 vertices.
(b) Draw a labelled diagram representing the 3 lines and the triangle's vertices.
(c) Find the triangle's area.
(d) Find the triangle's perimeter.

#2. Consider the rectangle with vertices $A = (2, 3)$, $B = (-1, 0)$, $C = (1, -2)$, and $D = (4, 1)$.
(a) Draw a labelled diagram. Label any intersections of the rectangle with the axes (you don't need to extend the sides of the rectangles and find the all the intersections of those lines with the axes).
(b) Find the rectangle's area.
(c) Find the rectangle's center.

#3. Consider the line L_1, which has equation $x + 2y = 6$.
(a) Find the equation of the line, L_2, which is parallel to L_1 and which passes through the point $(5, 0)$.
(b) Find the equation of the line, L_3, which is perpendicular to L_1 and which passes through the point $(-2, 1)$.
(c) Find the point of intersection of L_2 and L_3.
(d) For each of the three lines L_1, L_2, and L_3, find the intersections with the axes.
(e) Construct a diagram illustrating the lines and all the points of intersection found (both in (c) and in (d)).

#4. Consider the line L_1 with equation $x = -2$.
(a) Find the equation of a line which is parallel to L_1 and which passes through the point $(2, -4)$.
(b) Find the equation of a line which is perpendicular to L_1 and which passes through the point $(1, -3)$.

#5. Consider the function $f : [-2, 4] \to \mathbb{R}$ with $f(x) = -2x + 1$.
(a) Find the function's range.
(b) Is the function injective?

Practice for the Nov. 22nd quiz: solutions
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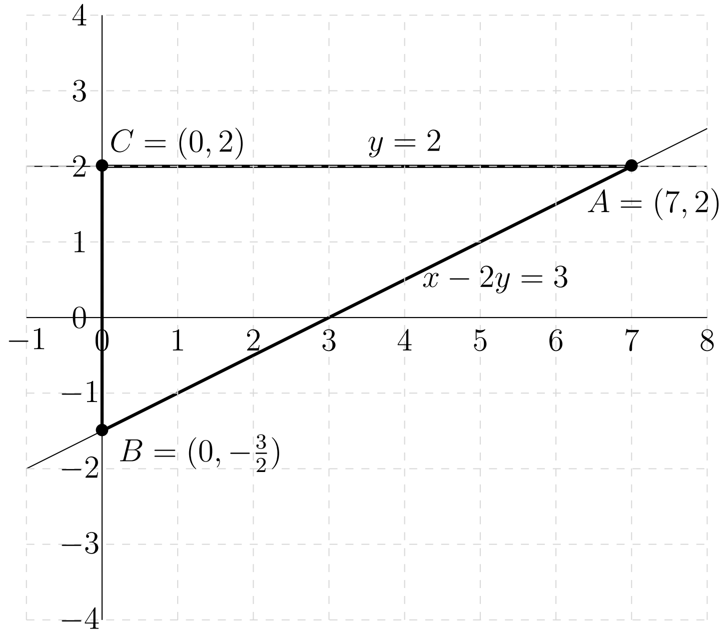
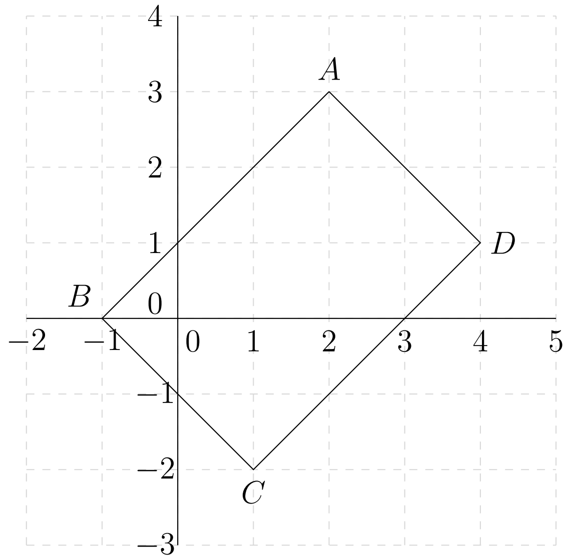
#1. Consider the triangle formed by the line $x - 2y = 3$, the line $y = 2$, and the y axis.
(a) Find the triangle's 3 vertices.
The vertices are the intersections of pairs of our lines.
Call the first vertex A, the intersection of $x - 2y = 3$ and $y = 2$. We can simply substitute $y = 2$ into the first line's equation, giving $x - 2(2) = 3$, so that $x = 7$. Therefore, $A = (7, 2)$.
Call the second vertex B, the intersection of $x - 2y = 3$ and the y axis. The y axis is where $x = 0$, so we substitute $x = 0$ into the first line's equation, giving $0 - 2y = 3$, so that $y = -\frac{3}{2}$. Therefore, $B = (0, -\frac{3}{2})$.
Call the second vertex C, the intersection of $y = 2$ and the y axis. On the y axis, we have $x = 0$, so the vertex will be $C = (0, 2)$.
(b) Draw a labelled diagram representing the 3 lines and the triangle's vertices.



(c) Find the triangle's area.
We can take the horizontal side as the base and the vertical side as the height, since it's a right-angled triangle. So the base is $7 - 0 = 7$ and the height is $2 - (-\frac{3}{2}) = \frac{7}{2}$. Therefore:
Area $= \frac{1}{2}(7)(\frac{7}{2}) = \frac{49}{4}$.
(d) Find the triangle's perimeter.
For the perimeter, we already have two of the sides, we just need the length of the hypoteneuse, which is $\sqrt{7^2 + (\frac{7}{2})^2} = 7\sqrt{\frac{5}{4}} = \frac{7}{2}\sqrt{5}$
So the perimeter is $7 + \frac{7}{2} + \frac{7}{2}\sqrt{2} = \frac{21+7\sqrt{5}}{2}$.

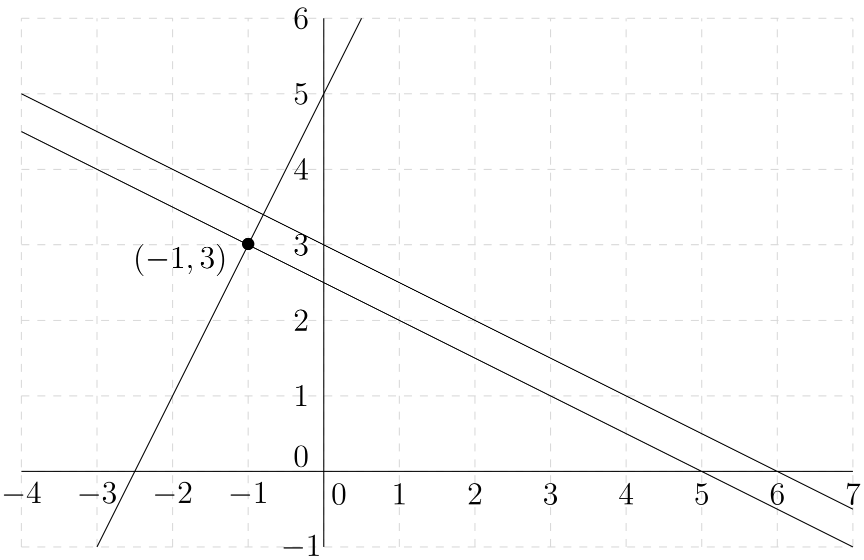
#2. Consider the rectangle with vertices $A = (2, 3)$, $B = (-1, 0)$, $C = (1, -2)$, and $D = (4, 1)$.
(a) Draw a labelled diagram. Label any intersections of the rectangle with the axes (you don't need to extend the sides of the rectangles and find the all the intersections of those lines with the axes).



(b) Find the rectangle's area.
We need the length and the width.
The length is the distance $AB = \sqrt{(2-(-1))^2 + (3-0)^2} = \sqrt{9+9} = 3\sqrt{2}$.
The width is the distance $BC = \sqrt{(1-(-1))^2 + (-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$.
Therefore the area is $(3\sqrt{2})(2\sqrt{2}) = 12$.
(c) Find the rectangle's center.
The center is the midpoint of opposite vertices. For example, we can take the midpoint of B and D.
The x coordinate will be $\frac{-1+4}{2} = \frac{3}{2}$.
The y coordinate will be $\frac{0+3}{2} = \frac{3}{2}$.
Therefore, the center is $(\frac{3}{2}, \frac{3}{2})$.

#3. Consider the line L_1, which has equation $x + 2y = 6$.
(a) Find the equation of the line, L_2, which is parallel to L_1 and which passes through the point $(5, 0)$.
L_1 can be rewritten as $y = -\frac{1}{2}x + 3$. So its gradient is $-\frac{1}{2}$. This means the grant of L_2 will also be $-\frac{1}{2}$, since they're parallel.
So L_2 will have an equation $y = -\frac{1}{2}x + b$. Substituting the point $(5, 0)$, we get:
$0 = -\frac{1}{2}(5) + b$
$\frac{5}{2} = b$
So L_2 will have the equation $y = -\frac{1}{2}x + \frac{5}{2}$.
(b) Find the equation of the line, L_3, which is perpendicular to L_1 and which passes through the point $(-2, 1)$.
Since L_3 is perpendicular, its gradient will be $-\dfrac{1}{-\frac{1}{2}} = 2$. So it's equation will be $y = 2x + b$ (not the same b as in the previous part, of course. Substituting in the point $(-2, 1)$, we get:
$1 = 2(-2) + b$
$5 = b$
So L_3 has equation $y = 2x + 5$.

(c) Find the point of intersection of L_2 and L_3.
To find the point of intersection, we solve the two equations simultaenously. We can substitute $y = 2x + 5$ from L_3 into the equation of L_1:
$2x + 5 = -\frac{1}{2}(x) + \frac{5}{2}$
$2x + \frac{1}{2}x = \frac{5}{2} - 5$
$\frac{5}{2}x = -\frac{5}{2}$
$x = -1$
Subtituting this back into L_3, we get $y = 2(-1) + 5 = 3$.
So the point of intersection is $(-1, 3)$.
(d) For each of the three lines L_1, L_2, and L_3, find the intersections with the axes.
L_1: When $x = 0$, $y = 3$. When $y = 0$, $x = 6$
L_2: When $x = 0$, $y = \frac{5}{2}$. When $y = 0$, $x = 5$
L_3: When $x = 0$, $y = 5$. When $y = 0$, $x = -\frac{5}{2}$
(e) Construct a diagram illustrating the lines and all the points of intersection found (both in (c) and in (d)).



#4. Consider the line L_1 with equation $x = -2$.
(a) Find the equation of a line which is parallel to L_1 and which passes through the point $(2, -4)$.
Since $x = -2$ is a vertical line, any line parallel to it will also be vertical. So it will have the form $x = c$. Now, since the point $(2, -4)$ is on the line we want, x will have to be 2. So the equation is $x = 2$.
(b) Find the equation of a line which is perpendicular to L_1 and which passes through the point $(1, -3)$.
A line perpendicular to it would be horizontal, so of the form $y = c$. Since it must pass through $(1, -3)$, the y value needs to be 3. So the equation is $y = -3$.

#5. Consider the function $f : [-2, 4] \to \mathbb{R}$ with $f(x) = -2x + 1$.
(a) Find the function's range.
To find the range we need to find the images of the domain's endpoints: -2 and 4.
$f(-2) = -2(-2) + 1 = 5$
$f(4) = -2(4) + 1 = -7$
So the range is the interval between these: $[-7, 5]$. The endpoints are included in the range since they were included in the domai.
(b) Is the function injective?
It is, since it's a diagonal line so it only goes through each y value once (or less).