Practice for the Oct. 11th quiz.
Mr. Neeman, 10A. October 7, 2011

Note: unless otherwise stated, you should always assume that a function's domain is its maximal domain and that its codomain is $\mathbb{R}$.

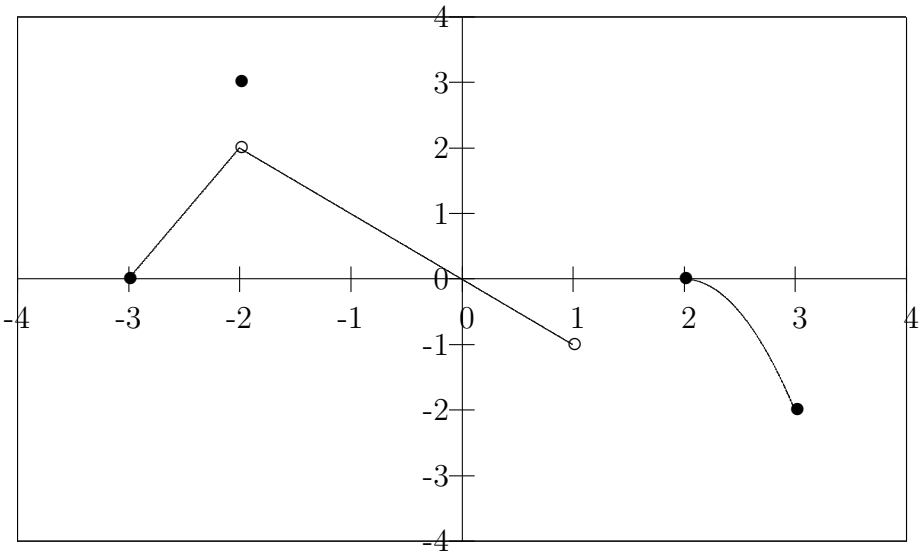
#1. Consider the function $f(x) = x^2 - 2x$.

(a) Find the image of 1.
(b) Find any preimages of 0.
(c) Sketch the function's graph (using a table of values). Label any intersections with the axes.
(d) Where is the function increasing and where is it decreasing?
(e) What is the function's domain?
(f) What is the function's range?
(g) What is the function's concavity?

#2. Consider the function $f(x) = 3 - x$.

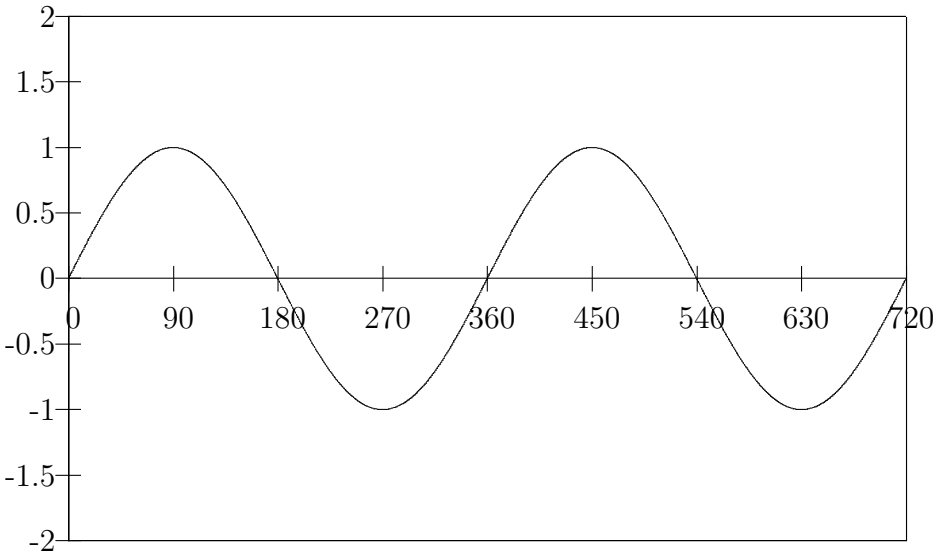
(a) Find the image of -2.
(b) Find any preimages of -1.
(c) Sketch the function's graph (using a table of values). Label any intersections with the axes.
(d) Is the function's increasing or decreasing?
(e) What is the function's domain?
(f) What is the function's range?

#3. Consider the function represented in the graph below.



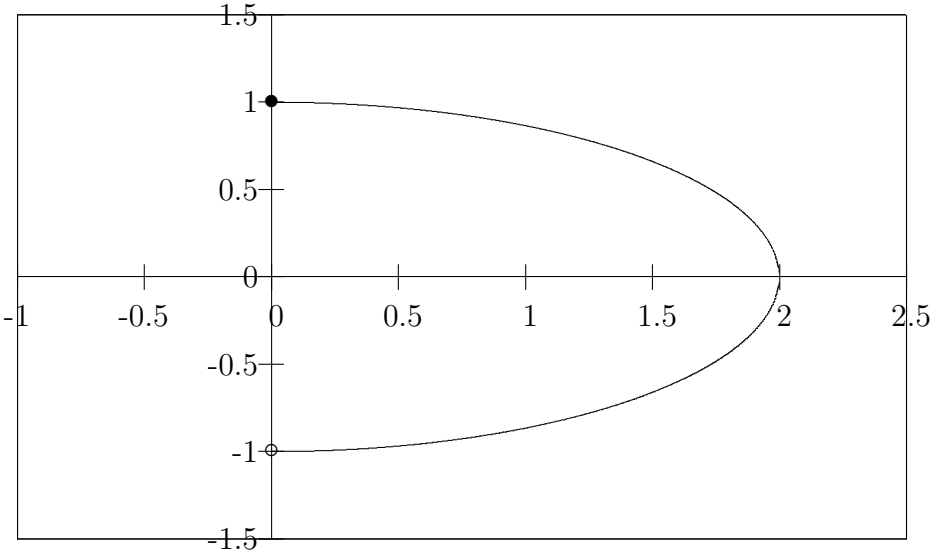
(a) Find the function's domain.
(b) Find the function's range.
(c) Find the image of -2.
(d) Find the image of -3.
(e) Find any preimages of 0.
(f) Where is the function strictly increasing?
(g) What is function's monotonicity on the interval [2,3]?

#4. Consider the function $f : [0, 720] \to \mathbb{R}$ with mapping $f(x) = \sin x$ (with x in degrees), and whose graph is shown below.



(a) Find the image of 270.
(b) Find any preimages of 1.
(c) Find any preimages of -1.5.
(d) What is the function's range?

#5. Could the following be the graph of a function?



Solutions

#1. Consider the function $f(x) = x^2 - 2x$.

(a) Find the image of 1.

$f(1) = 1^2 - 2(1) = -1$

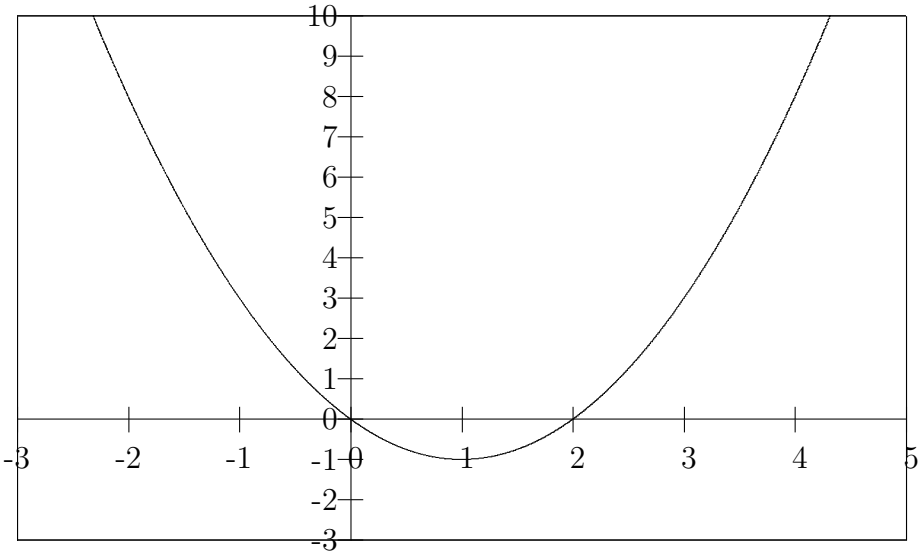
(b) Find any preimages of 0.

$0 = x^2 - 2x$

$0 = x(x - 2)$

So $x = 0$ or $x = 2$. So the preimages are 0 and 2.

(c) Sketch the function's graph (using a table of values). Label any intersections with the axes.



Notice that, if we complete the square, we get $f(x) = (x - 1)^2 - 1$. This can help us figure out what the graph should look like (it will have its lowest point when $x - 1 = 0$, which is when $x = 1$, which gives $f(1) = -1$.

(d) Where is the function increasing and where is it decreasing?

It in increasing on $[1, \infty[$ and decreasing on $]-\infty, 1]$.

(e) What is the function's domain?

$\mathbb{R}$

(f) What is the function's range?

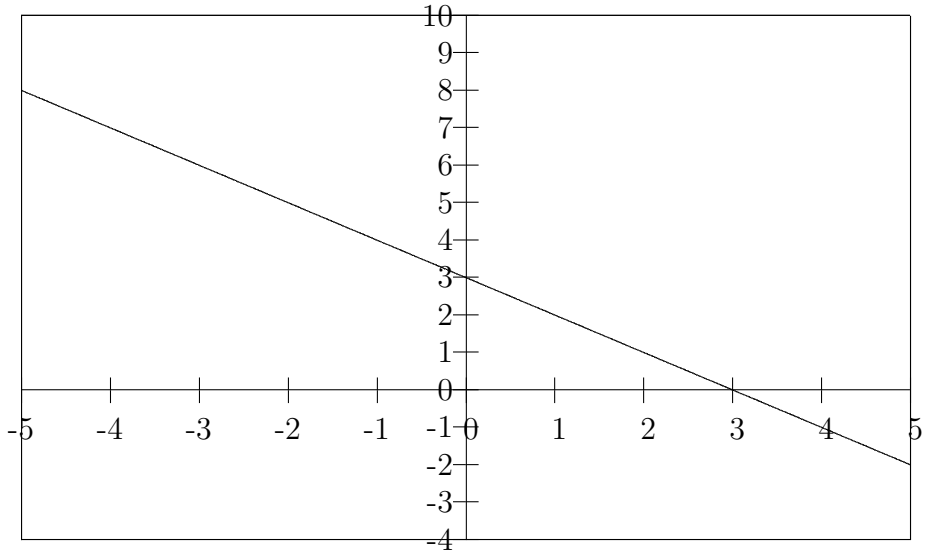
$[-1, \infty[$

(g) What is the function's concavity?

Concave up.

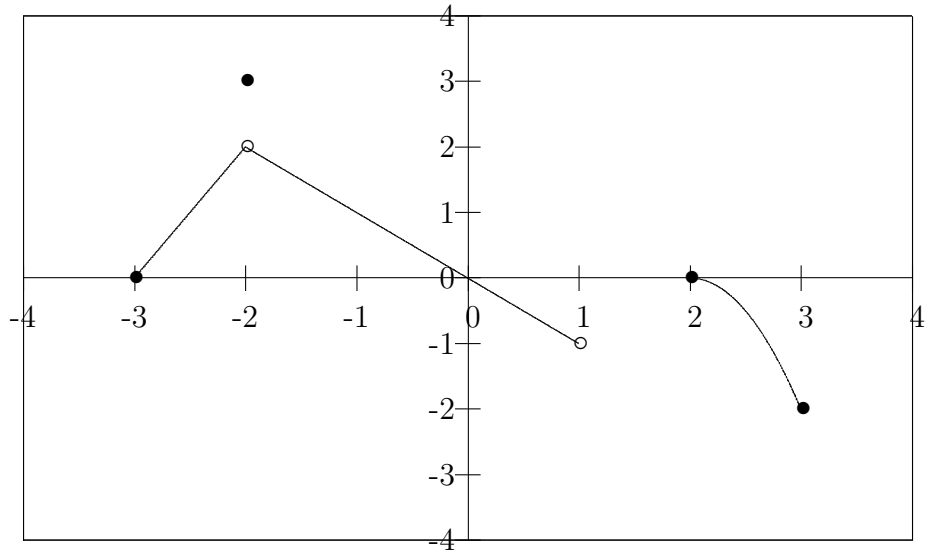
#2. Consider the function $f(x) = 3 - x$.

(a) Find the image of -2.
$f(-2) = 3 - (-2) = 5$
(b) Find any preimages of -1.
$-1 = 3 - x$
$x = 3 + 1 = 4$
So there's one preimage: 4.
(c) Sketch the function's graph (using a table of values). Label any intersections with the axes.



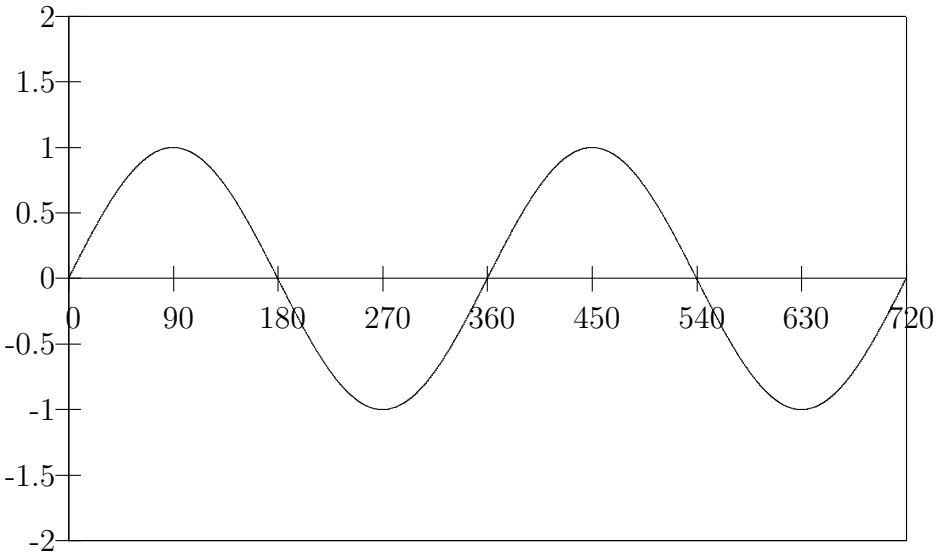
(d) Is the function's increasing or decreasing?
Decreasing
(e) What is the function's domain?
$\mathbb{R}$
(f) What is the function's range?
$\mathbb{R}$

#3. Consider the function represented in the graph below.



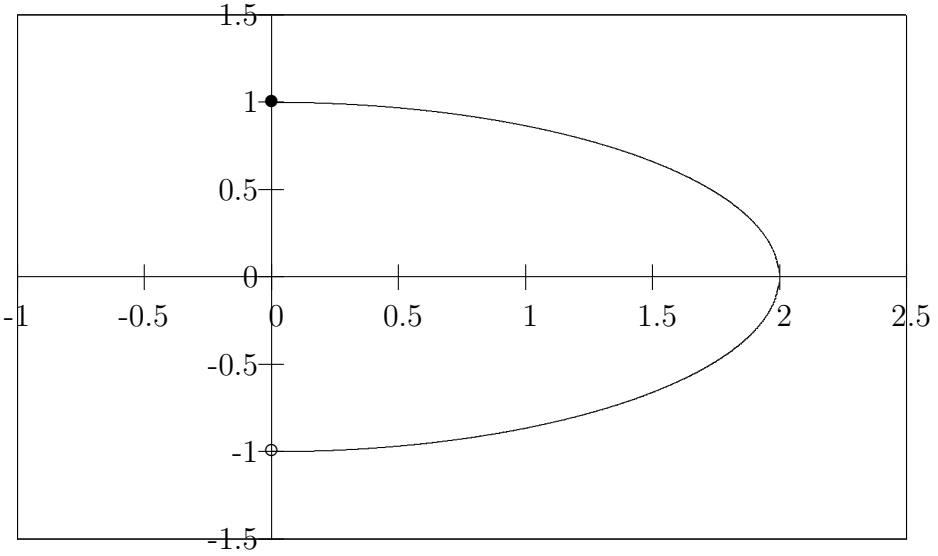
(a) Find the function's domain.
$[-3, 1[\cup[2, 3]$
(b) Find the function's range.
$[-2, 2[\cup\{3\}$
(c) Find the image of -2.
3
(d) Find the image of -3.
0
(e) Find any preimages of 0.
-3, 0, and 2.
(f) Where is the function strictly increasing?
$[-3, -2]$. -2 is included, but if you were to write $[-3, -2[$ that would be good enough for me.
(g) What is function's monotonicity on the interval [2,3]?
Strictly decreasing (also, decreasing).

#4. Consider the function $f : [0, 720] \to \mathbb{R}$ with mapping $f(x) = \sin x$ (with x in degrees), and whose graph is shown below.



(a) Find the image of 270.
-1
(b) Find any preimages of 1.
90 and 450.
(c) Find any preimages of -1.5.
There aren't any.
(d) What is the function's range?
$[-1, 1]$

#5. Could the following be the graph of a function?



No, because some points would have two images (any point in $]0, 2[$ since there a vertical line would intersect the curve twice).