Functions on finite sets
Mr. Neeman, 10A. October 22, 2011

When we say "functions on finite sets," we really just mean functions with finite domains. The reason for this way of phrasing it is to avoid confusing "functions on finite sets" with "functions on finite intervals." So let's look at this distinction:

A **finite set** is a set with a finite number of elements. For example: $A = \{0, 2, 5\}, B = \{a, c, d\}$, etc. are finite sets.
A **finite interval** is an interval whose endpoints are finite real numbers. For example $[-4, 3]$, $[0, 10]$, etc. are finite intervals.

Note, however, that **a finite interval is actually not a finite set**, since there are infinitely many numbers in any non-trivial interval (a trivial interval would be something like $[2, 2]$, whose two endpoints are the same).

So when we speak of functions on finite sets, we means functions whose domain has a finite number of elements. These are typically not very interesting, and mainly appear as toy examples. But they are functions and most of the usual function concepts apply. Note that their codomain might be a finite set or it might not be (it could be $\mathbb{R}$, for example), though we will only be looking at ones with finite codomains.

E.g. #1. Suppose $f : \{1, 3, 4, 8\} \to \{0, 1\}$, and suppose $f(1) = 0, f(3) = 0, f(4) = 1, f(8) = 0$.
(a) Find the image of 3.
This is trivial: we know $f(3) = 0$, so 0 is the image of 3.
(b) Find any preimages of 0.
We just have to look to see which elements of the domain have 0 as their image. These are 1, 3, and 8.
(c) What is its range?
The range is just the set of images (the set of elements of the codomain which have preimages). So it's $\{0, 1\}$
(d) Is the function injective?
It's not, because 0 has more than one preimage.
(e) Is the function surjective?
It is, because the codomain contains two elements, each of which has at least one preimage, so every element of the codomain has a preimage.
(f) Is the function bijective?
It's not, since it's not injective.

E.g. #2. Suppose $f : \{a, b, c, d\} \to \{a, b, d, g\}$, and suppose $f(a) = b, f(b) = d, f(c) = a, f(d) = g$.
(b) Find any preimages of d.
We just have to look to see which elements of the domain have d as their image. The only one is b.
(c) Is the function injective? Is it surjective? Is it bijective?
In fact, we can see that each of the elements of the codomain has exactly one preimage. This means the function is injective, surjective, and bijective.

E.g. #3. Suppose $f : \{a, b, c\} \to \{-2, 0, 1, 3, 7\}$, and suppose $f(a) = 1, f(b) = 0, f(c) = 7$.
(a) Find any preimages of 3.
There aren't any.
(b) What is the function's range?
The range is just the set of images (the set of elements of the codomain which have preimages). So it's $\{1, 0, 7\}$.
(c) Is the function injective?
It is, because some elements of the codomain have one image, others have none, but none have more than one.
(d) Is the function surjective?
No, some elements of the codomain (e.g. -2) don't have any preimages.
(e) Is the function bijective?
No, since it's not surjective.

<u>Practice exercises</u>

#1. Suppose $f : \{0, 5, -2, -5\} \to \{0, 1\}$, and suppose $f(0) = 0, f(5) = 1, f(-2) = 0, f(-5) = 1$.
(a) Find the image of 5.
(b) Find any preimages of 0.
(c) What is the function's range?
(c) Is the function injective? Is it surjective? Is it bijective?

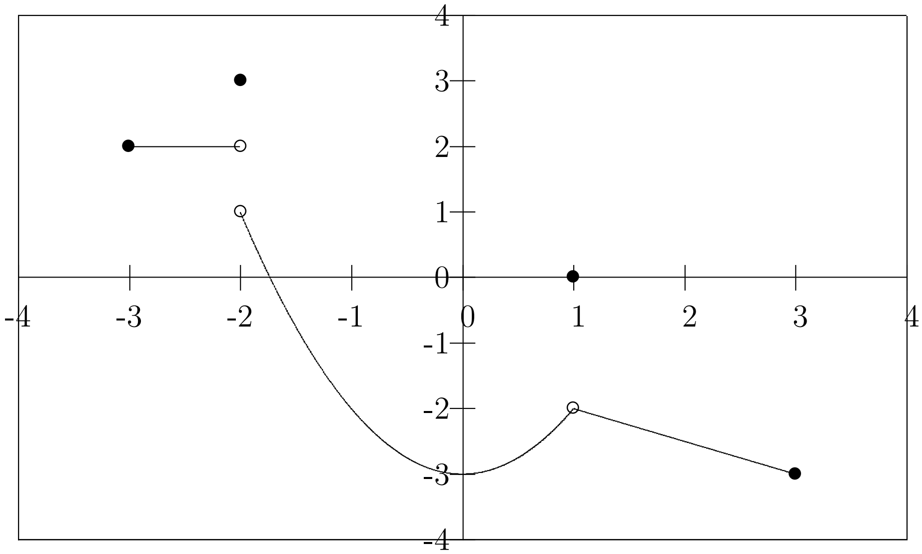
#2. Suppose $f : \{1, 5, -7, \frac{1}{2}\} \to \{a, d, g, h\}$, and suppose $f(1) = d, f(5) = d, f(-7) = a, f(\frac{1}{2}) = h$.
(a) Find any preimages of d.
(b) Find any preimages of h.
(c) What is the function's range?
(c) Is the function injective? Is it surjective? Is it bijective?

<u>Homework</u>

#1. Suppose $f : \{1, 2, 3, 6\} \to \{1, 2, 3\}$, and suppose $f(1) = 2, f(2) = 2, f(1) = 3, f(6) = 1$.
(a) Find the image of 2.
(b) Find any preimages of 2.
(c) What is the function's range?
(c) Is the function injective? Is it surjective? Is it bijective?

#2. Suppose $f : \{a, b, f, h\} \to \{1, 4, 5, 8\}$, and suppose $f(a) = 5, f(b) = 4, f(f) = 1, f(h) = 8$.
(a) Find any preimages of 4.
(c) What is the function's range?
(c) Is the function injective? Is it surjective? Is it bijective?

#3. Consider the function shown in the graph. Suppose its codomain is $\mathbb{R}$.



(a) What is the function's domain?
(b) What is the function's range?
(c) Is the function injective?
(d) Is the function surjective?
(e) Is the function bijective?
(f) Find any preimages of 3.
(g) Find the image 0.
(h) Find the image of 1.
(i) What is the function's monotonicity on the interval $]-2,0]$?
(j) What is the function's curvature on the interval $]-2,0]$?

Solutions for practice exercises

#1. Suppose $f : \{0, 5, -2, -5\} \to \{0, 1\}$, and suppose $f(0) = 0, f(5) = 1, f(-2) = 0, f(-5) = 1$.
(a) Find the image of 5.
It's 1.
(b) Find any preimages of 0.
They are 0 and -2.
(c) What is the function's range?
It's $\{0, 1\}$.
(c) Is the function injective? Is it surjective? Is it bijective?
The function is not injective or bijective, but it is surjective.

#2. Suppose $f : \{1, 5, -7, \frac{1}{2}\} \to \{a, d, g, h\}$, and suppose $f(1) = d, f(5) = d, f(-7) = a, f(\frac{1}{2}) = h$.
(a) Find any preimages of d.
They are 1 and 5.
(b) Find any preimages of h.
The only preimage is $\frac{1}{2}$.
(c) What is the function's range?
The range is $\{a, d, h\}$
(c) Is the function injective? Is it surjective? Is it bijective?
It's not injective, surjective, or bijective.