Practice for Oct. 21st quiz

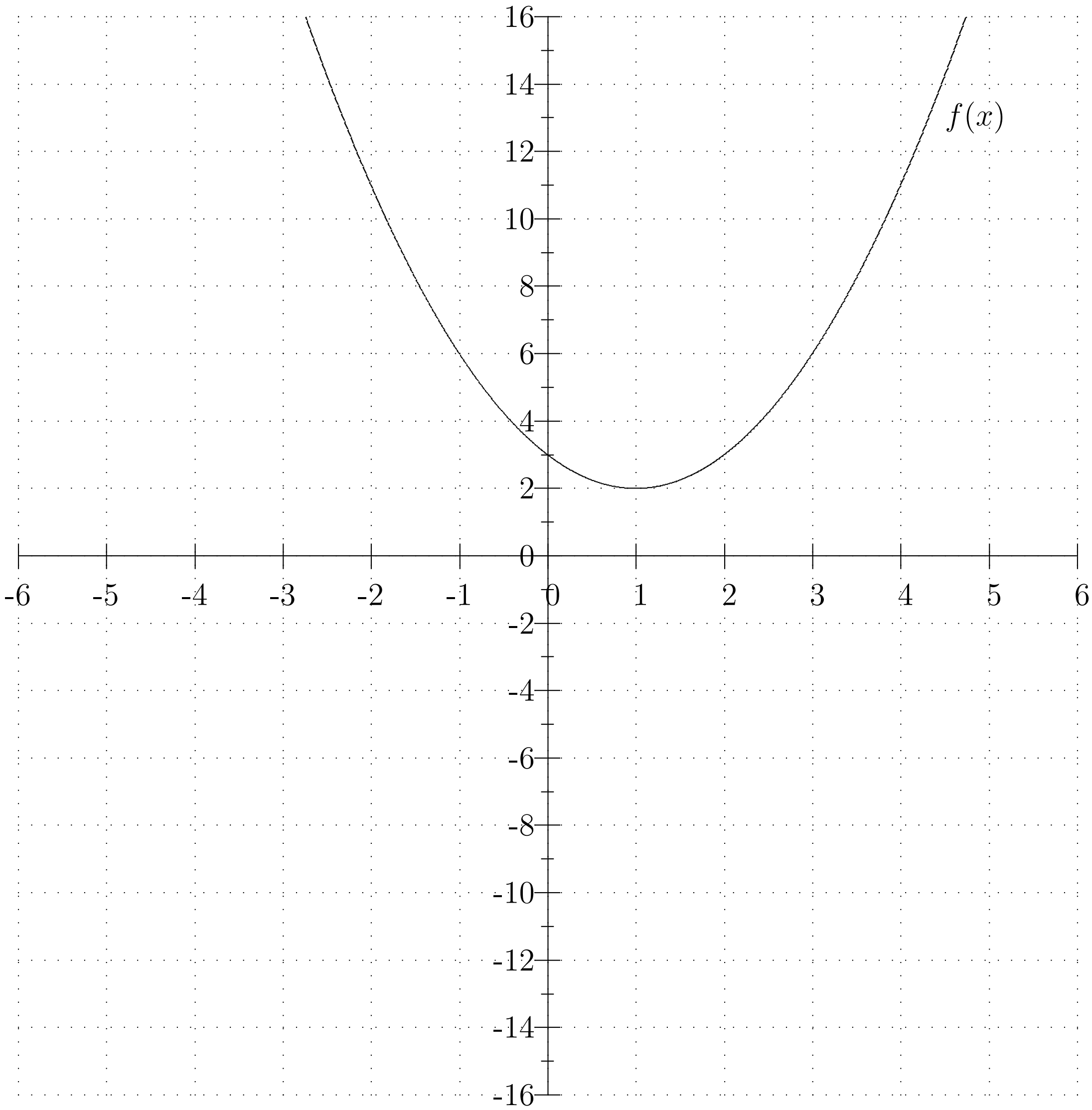
Mr. Neeman, 10A. October 18, 2011

#1. Consider the function $f(x) = 3 - x^2$. Find:
(a) $f(-1)$
(b) $f(2-y)$
(c) $f(x^2)$
(d) $f(x+2)$

#2. Consider the functions $f(x) = \frac{1}{x}$ and $g(x) = x - 3$. Find:
(a) $fg(x)$
(b) $fg(1)$
(c) $fg(y+3)$
(d) $gf(x)$
(e) $gf(x^2)$
(f) $-gf(-x)$

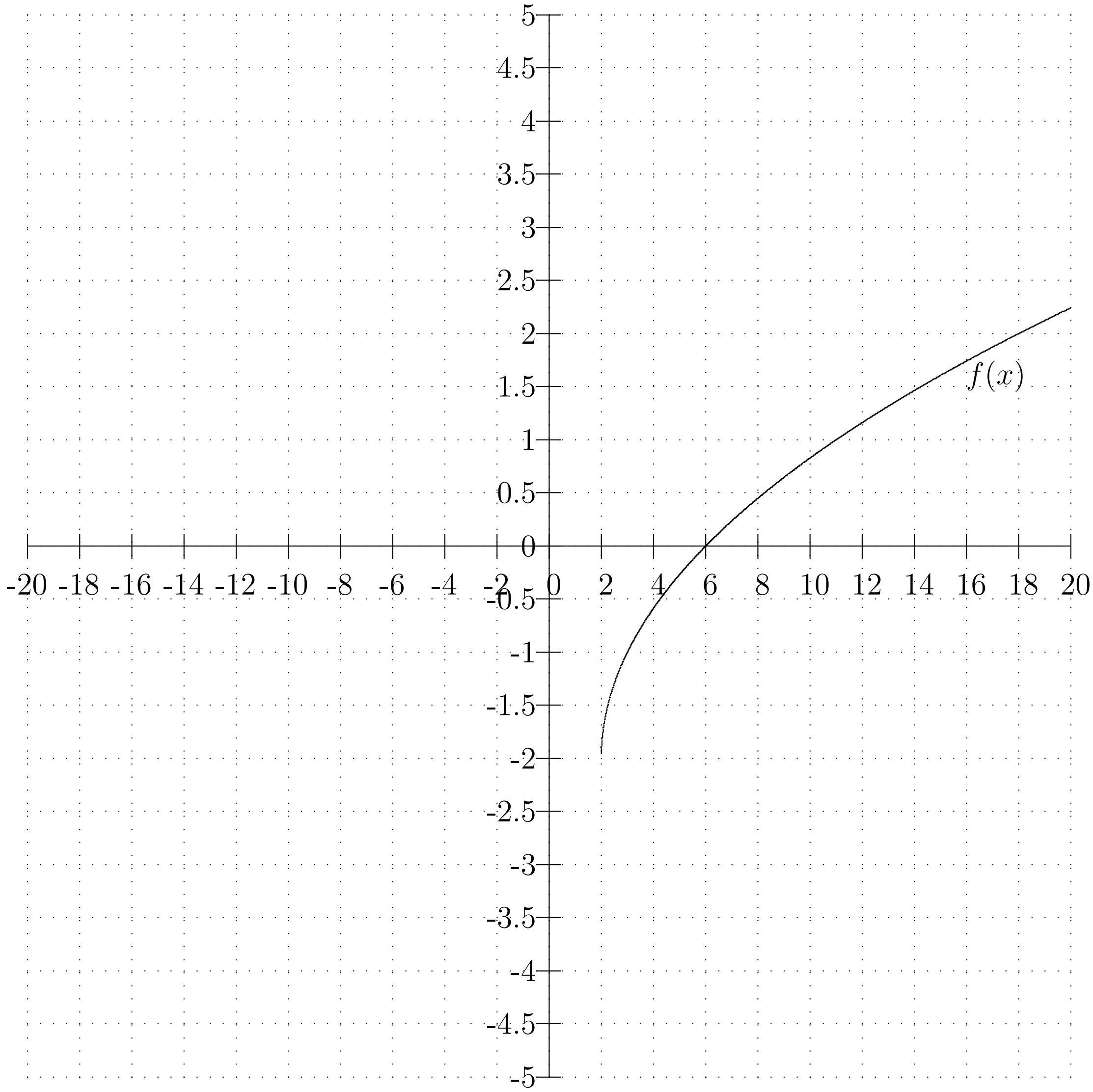
#3. Describe, in words, what the graph of each of the following functions looks like, compared to the graph of $f(x)$.
(a) $f(x-2)$
(b) $-f(x)$
(c) $\frac{1}{2}f(x)$
(e) $f(-x) + 3$
(f) $-f(x+1)$
(g) $f(3x)$

#4. The diagram below shows the graph of a function, $f(x)$.



(a) Is the function injective?
(b) Is the function surjective?
(c) Is the function bijective?
(d) Sketch and the label the graph of $-f(x)$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
(e) Sketch and the label the graph of $-f(x+5)$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
(f) Sketch and the label the graph of $f(x)+4$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.

#5. The diagram below shows the graph of a function, $f(x)$. Its domain is $[2, \infty[$.



(a) Is the function injective?
(b) Is the function surjective?
(c) Is the function bijective?
(d) Sketch and the label the graph of $f(\frac{x}{2})$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
(e) Sketch and the label the graph of $f(-x)$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
(f) Sketch and the label the graph of $f(-x)+3$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.

Solutions

#1. Consider the function $f(x) = 3 - x^2$.

(a) $f(-1)$
$= 3 - (-1)^2$
$= 3 - 1$
$= 2$

(b) $f(2 - y)$
$= 3 - (2 - y)^2$

(c) $f(x^2)$
$= 3 - (x^2)^2$
$= 3 - x^4$

(d) $f(x + 2)$
$= 3 - (x + 2)^2$

#2. Consider the functions $f(x) = \frac{1}{x}$ and $g(x) = x - 3$. Find:

(a) $fg(x)$
$= f(g(x))$
$= f(x - 3)$
$= \frac{1}{x-3}$

(b) $fg(1)$
$= \frac{1}{1-3}$ (using (a))
$= -\frac{1}{2}$

(c) $fg(y + 3)$
$= \frac{1}{(y+3)-3}$ (using (a))
$= \frac{1}{y}$

(d) $gf(x)$
$= g(f(x))$
$= g\left(\frac{1}{x}\right)$
$= \frac{1}{x} - 3$

(e) $gf(x^2)$
$= \frac{1}{x^2} - 3$ (using (d))

(f) $-gf(-x)$
$= -\left(\frac{1}{-x} - 3\right)$ (using (d))
$= \frac{1}{x} + 3$

#3. Describe, in words, what the graph of each of the following functions looks like, compared to the graph of $f(x)$.

(a) $f(x-2)$
$f(x)$ translated by two units to the right.

(b) $-f(x)$
$f(x)$ reflected along the x axis.

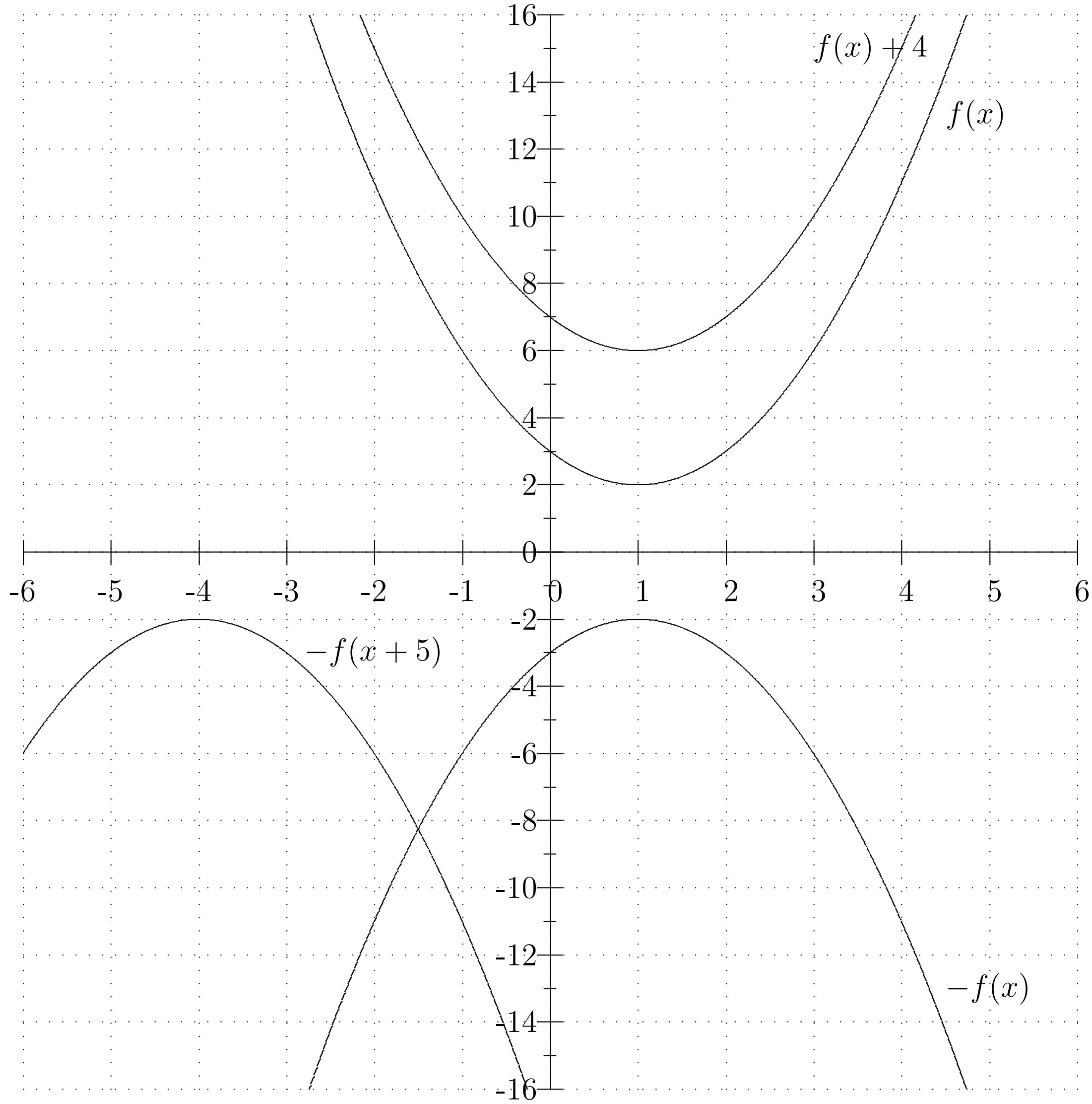
(c) $\frac{1}{2}f(x)$
$f(x)$ squeezed vertically by a factor of 2, around the x axis.
(Also, $f(x)$ scaled vertically by a factor of $\frac{1}{2}$, around the x axis.)

(e) $f(-x)+3$
$f(x)$ reflected along the y axis and then translated by three units upwards.
(Also, $f(x)$ translated by three units upwards and then reflected along the y axis. In this case the result is the same regardless of the order of the transformations.)

(f) $-f(x+1)$
$f(x)$ translated by 1 unit to the left and then reflected along the x axis.
(Here, too, the result is the same if you first reflect along the x axis and then translate by 1 unit to the left.)

(g) $f(3x)$
$f(x)$ squeezed horizontally by a factor of 3, around the y axis.

#4.



(a) Is the function injective? No.

(b) Is the function surjective? No.

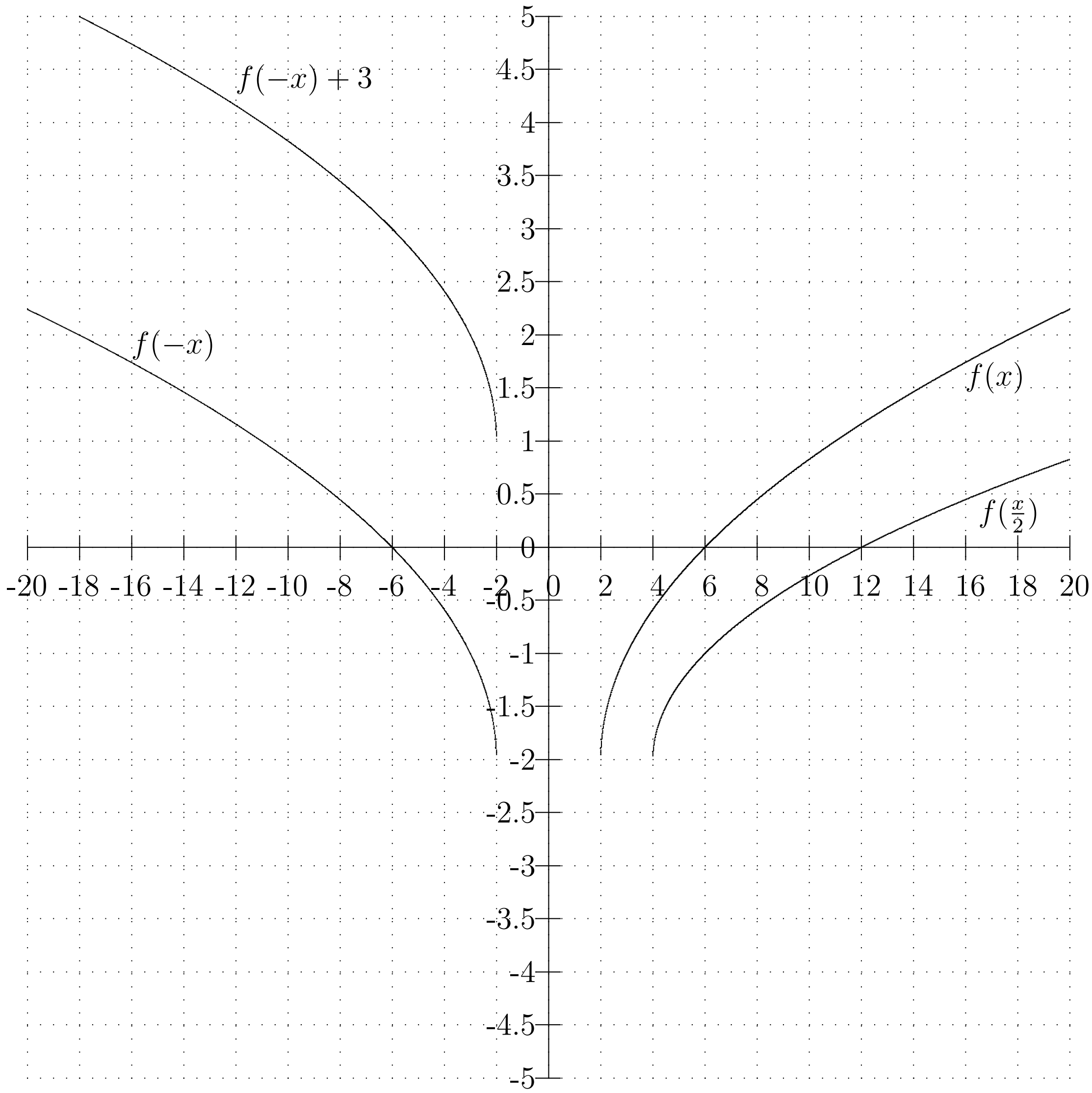
(c) Is the function bijective? No.

(d) Sketch and the label the graph of $-f(x)$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
If $x = 3$: $-f(x) = -f(3) = -6$, which fits the graph.

(e) Sketch and the label the graph of $-f(x+5)$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
If $x = -2$: $-f(x+5) = -f(-2+5) = -f(3) = -6$, which fits the graph.

(f) Sketch and the label the graph of $f(x)+4$. Check your answer by subtituting a value of x and seeing whether the answer fits your sketch.
If $x = 3$, $f(x) + 4 = f(3) + 4 = 6 + 4 = 10$, which fits the graph.

#5. The domain of f is $[2, \infty[$



(a) Is the function injective? Yes.
(b) Is the function surjective? No.
(c) Is the function bijective? No.

(d) $f(\frac{x}{2})$.
With $x = 6$: $f(\frac{x}{2}) = f(\frac{6}{2}) = f(3) = -1$, which is ok. Notice the domain of $f(\frac{x}{2})$ is $[4, \infty[$.

(e) $f(-x)$.
With $x = -18$: $f(-x) = f(-(-18)) = f(18) = 2$, which is ok. Notice the domain of $f(-x)$ is $]-\infty, -2]$.

(f) $f(-x) + 3$.
With $x = -18$: $f(-x) + 3 = f(-(-18)) + 3 = f(18) + 3 = 2 + 3 = 5$, which is ok. Notice the domain of $f(-x) + 3$ is $]-\infty, -2]$.