

Mathematics Department 1st bimester, FS 2011
 Mr. Neeman Total points possible: 20
 Quiz #3, 10A Solutions
 Topic: Inequalities Estimated time: 40 minutes

Instructions: Answer all the questions, working carefully and showing your work neatly. Make sure the final answer is easily identifiable.

#1. Solve the inequality $x^3 + x^2 < x + 1$. (5 pts)

$$x^3 + x^2 - x - 1 < 0$$

$$x^2(x + 1) - (x + 1) < 0$$

$$(x^2 - 1)(x + 1) < 0$$

$$(x + 1)^2(x - 1) < 0$$

The key values are $x = -1$ and $x = 1$. We do the table. Note that the last row is the product of the second and third rows.

	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
$x + 1$	-	0	+	+	+
$(x + 1)^2$	+	0	+	+	+
$x - 1$	-	-	-	0	+
$(x + 1)^2(x - 1)$	-	0	-	0	+

We're looking for it to be negative, so it's $x < -1$ and $-1 < x < 1$.

#2. Solve the inequality $x^2 + 2x - 8 \geq 0$. (4 pts)

$$(x + 4)(x - 2) \geq 0$$

The key values are $x = -4$ and $x = 2$. We do the table.

	$x < -4$	$x = -4$	$-4 < x < 2$	$x = 2$	$x > 2$
$x + 4$	-	0	+	+	+
$x - 2$	-	-	-	0	+
$(x + 4)(x - 2)$	+	0	-	0	+

We're looking for it to be 0 or positive, so we get $x \leq -4$ and $x \geq 2$.

#3. Solve the inequality $(x + 3)(x^2 + 7) \geq 0$. (3 pts)

$x^2 + 7$ is an irreducible quadratic. This means it has no roots, since for it to be zero we would need $x^2 = -7$ which can't happen, whatever x is. So we know $x^2 + 7 \neq 0$, and we can divide both sides by it. We get:

$$x + 3 \geq 0$$

$$x \geq -3.$$

#4. Solve the inequality $2\frac{x+1}{x-1} \leq -x + \frac{x^2}{x-1}$. (4 pts)

$$\frac{2x+2}{x-1} + x - \frac{x^2}{x-1} \leq 0$$

$$\frac{2x+2+x^2-x-x^2}{x-1} \leq 0$$

$$\frac{x+2}{x-1} \leq 0$$

Our key values are $x = -2$ and $x = 1$. We can do the table. Note the last row is the product of the first and third rows.

	$x < -2$	$x = -2$	$-2 < x < 1$	$x = 1$	$x > 1$
$x + 2$	-	0	+	+	+
$x - 1$	-	-	-	0	+
$\frac{1}{x-1}$	-	-	-	undef.	+
$\frac{x+2}{x-1}$	+	0	-	undef.	+

We're looking for it to be 0 or negative, so we get $-2 \leq x < 1$.

#5. Solve the inequality $2x^2 + 20x + 50 > 0$. (4 pts)

We can divide both sides by 2. Since 2 is positive, the inequality sign stays the same.

$$x^2 + 10x + 25 > 0$$

$$(x + 5)^2 > 0$$

The key value is $x = -5$. We can do the table:

	$x < -5$	$x = 5$	$x > 5$
$(x + 5)$	-	0	+
$(x + 5)^2$	+	0	+

We're looking for it to be positive. So we get $x < -5$ and $x > 5$.